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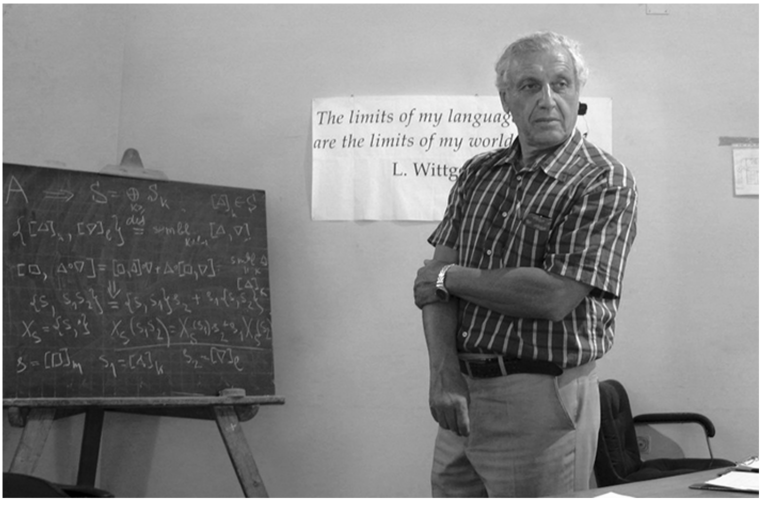
Alexandre Mikhailovich Vinogradov (obituary)



On 20 September 2019, Alexandre Mikhailovich Vinogradov, a remarkable mathematician and an extraordinary person, passed away.

He was born on 18 February 1938 in Novorossiysk. During World War II he and his mother were evacuated to Kungur (his father served in the army), and later his parents settled in Kuntsevo, at that time not yet a part of Moscow. In 1955 he enrolled in the Faculty of Mechanics and Mathematics at Moscow State University, and in 1960 he became a graduate student. After defending his Ph.D. thesis in 1964, he taught students at the Moscow Mining Institute for a year. Then N. V. Efimov, who was then the dean, invited Vinogradov to work in the Department of Higher Geometry and Topology of the Faculty of Mechanics and Mathematics (P. S. Alexandrov was head of the department at that time), where he worked until his departure to Italy in 1990. He became a doctor of the physical and mathematical sciences in 1984. In 1993–2010 he was a professor at the University of Salerno (Italy).

When Vinogradov was only a second-year undergraduate student, he published two papers on number theory (jointly with B. N. Delaunay and D. B. Fuchs). However, towards the end of his term at the university his interests changed: in his senior year and in graduate school he began to study algebraic topology in the seminar of A. S. Schwarz (Shvarts). His Ph.D. thesis, under the formal supervision of V. G. Boltyansky, was on the homotopy properties of the space of embeddings of

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a circle in a sphere or a ball. One of his first papers (1958) concerned the Adams spectral sequence, which at that time was rightly considered the top of algebraic topology. In his note [1], he announced a solution of a problem of F. Adams on the connection between higher cohomological operations and the Adams filtration of stable homotopy groups of spheres, and Adams wrote a favourable review of the note. Vinogradov worked on problems in algebraic and differential topology up until the early 1970s. From 1967 he led a research seminar on the same subject.

For the second and last time, Vinogradov radically changed the area of his mathematical activities at the turn of the decade 1960–1970. Inspired by ideas of Sophus Lie, he began to think through the foundations of the geometric theory of partial differential equations. After reviewing the work of D. Spencer, H. Goldschmidt and D. Quillen on formal solvability, he embarked on an investigation of the algebraic and, in particular, cohomological aspects of solvability.

In 1972, his short article [2] "The logic algebra for the theory of linear differential operators" appeared in *Doklady Akademii Nauk SSSR*[1] (it was not easy to publish lengthy texts in those years). There he constructed the so-called basic functors of differential calculus in commutative algebras. On just four pages of the journal text he showed in an elegant manner that the category of modules over a commutative algebra with unit is sufficient to define and study the fundamental properties of concepts such as vector fields, differential forms, jets, linear differential operators, and so on, and that their geometric prototypes arise if we choose the algebra of smooth functions on a manifold as the algebra, and the spaces of sections of vector bundles over the manifold as modules. Later, an extended version of this article became the first chapter of the book [3] and was partially included in [4]. A modernised version of the theory was published recently in [5], and applications of it to the construction of an algebraic model of Hamiltonian mechanics were considered in [6].

We should note here that Vinogradov was a natural 'mathematical polyglot': he easily switched from the language of algebra to that of differential geometry and often used a 'dictionary' for (sometimes not at all trivial) translations of statements familiar in classical differential geometry into the language of geometry of infinitely extended equations. Such multilingualism helped him to produce new meaningful constructions, definitions, and statements.[2] He was always attracted by the possibility of an invariant, coordinate-free (and, hence, elegant) exposition, be it in Hamiltonian mechanics [8] or geometry [9].

His approach to non-linear differential equations as geometric objects with a general theory and applications, was described in detail in the monographs [3] and [10], as well as the papers [11] and [12]. He combined infinitely extended equations into a category [13] whose objects he called *diffieties* (differential varieties), and for studying diffieties he developed a theory which became known as *secondary calculus*[3] [14], [15]. Central to this theory is the $\mathscr{C}$-spectral sequence (the Vinogradov spectral sequence), which was announced in [16] and later described in detail in [17].

[1] Partly translated into English as *Soviet Mathematics. Doklady*.

[2] For example, in this way he came to the concept of a differential cover [7], which is central to the non-local geometry of differential equations and turns out to be extremely important for understanding a number of structures related to integrable systems.

[3] By analogy with secondary quantisation.

The E_1-term of this spectral sequence gives a uniform cohomological approach to many previously scattered concepts and statements, including Lagrangian formalism with constraints, conservation laws, cosymmetries, Noether's theorem, and the Helmholtz criterion in the inverse problem of the calculus of variations (for arbitrary non-linear differential operators), greatly expanding the understanding of these classical statements. A special case of the $\mathscr{C}$-spectral sequence (corresponding to the 'empty' equation, that is, the space of infinite jets) is the so-called variational bicomplex.

The results of [16] were subsequently generalised by R. L. Bryant and P. A. Griffiths [18] (who used the language of exterior differential systems), A. M. Verbovetsky [19] (using the horizontal de Rham complex), and T. Tsujishita [20]. The ideas underlying the construction of the $\mathscr{C}$-spectral sequence and the results naturally arising from these ideas were the first steps towards what is now called cohomological physics (for example, see [21] by J. Stasheff).

The important papers [22] and [23] also belong to this area. In the first, Vinogradov constructed a new bracket on the graded algebra of linear transformations of a cochain complex. The Vinogradov bracket (which he called the L-commutator) is skew-symmetric and satisfies the Jacobi identity up to a coboundary. His construction anticipated the general concept of a derived bracket on the Loday differential algebra (or Leibniz algebra), introduced by Y. Kosmann-Schwarzbach in 1996; see [24]. The Vinogradov bracket is a skew-symmetric version of a derived bracket constructed from the coboundary operator. Derived brackets and their generalisations play an extremely important role in modern applications of homotopy Lie algebras and Lie algebroids. Vinogradov's paper [22] was pioneering in this area.

In particular, Vinogradov showed that the classical Schouten bracket (on multivector fields) and Nijenhuis bracket (on vector fields with coefficients in differential forms) are the restrictions of his bracket to the corresponding subalgebras of superdifferential operators on the exterior algebra of forms. In a subsequent joint paper with A. Cabras [23], he applied these results to Poisson geometry by constructing new examples of derived differential geometric brackets.

Modern developments lead to generalisations of Lie (super)algebras with 'higher brackets' (that is, brackets with $n > 2$ arguments). Such generalisations include the strongly homotopy Lie algebras of Lada and Stasheff (also known as L_∞-algebras) and 'Filippov algebras'. These structures were analysed and compared in the papers [25]–[27] by Vinogradov and his coauthors.

It should be noted that Vinogradov's scientific interests were always very strongly motivated by complex and important problems of modern physics, from the dynamics of sound beams [28] to the magnetohydrodynamic equations (the so-called Kadomtsev–Pogutse equations used in the theory of stability of high-temperature plasma in tokamaks) [29]. The mathematical understanding of the fundamental physical concept of the observable received much attention in the book [4], written by Vinogradov in collaboration with the participants in his seminar and published under the pseudonym Jet Nestruev.

Whatever Vinogradov was occupied with — geometry of equations, the Schouten and Nijenhuis brackets [22], [23], mathematical problems in gravity theory [30]–[32], n-ary generalisations of Lie algebras [25]–[27], or structural analysis of the latter [33], [34], — all his works were distinguished by an unorthodox approach, great depth, and non-triviality of the results. His printed heritage consists of more than a hundred papers and ten monographs.

His scientific activity was not limited to work in his office. For many years, he led a research seminar at Moscow State University, consisting of two parts, mathematics and physics. This seminar became a prominent feature in the mathematical life of Moscow from the 1960s to the 1980s. He brought up a galaxy of students (in Russia, Italy, Switzerland, Poland), 19 of whom received Ph.D. degrees, 6 received D.Sc. degrees, and one was elected a corresponding member of the Russian Academy of Sciences. On his initiative and under his leadership several Diffiety Schools were held in Italy, Russia, and Poland. He was the soul of the series of chamber conferences "Current Geometry" held in Italy from 2000 to 2010, as well as the comprehensive Moscow conference "Secondary Differential Calculus and Cohomological Physics" (1997), with proceedings published in [15].

He was one of the initiators and an active participant in the creation of the Erwin Schrödinger International Institute for Mathematics and Physics (ESI) in Vienna, as well as the *Journal of Differential Geometry and its Applications*, where he was a member of the editorial board until his last days. In 1985 Vinogradov founded a laboratory for the study of various aspects of the geometry of differential equations at the Institute of Software Systems in Pereslavl-Zalessky and was its scientific supervisor until his departure for Italy. He also lectured to students who were not accepted by the Faculty of Mechanics and Mathematics of Moscow State University because of their Jewish origins (Vinogradov called this 'the Peoples' Friendship University').

He was a very versatile person: he played violin, wrote poetry in Italian, played for the faculty water polo team, and was an avid soccer player. However, the main thing for him was certainly mathematics.

Alexander Mikhailovich Vinogradov continues to live in his works, and in the memory of students, family, and friends.

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