Workshop on

Integrable Nonlinear Equations

Mikulov, Czech Republic

October 18–24, 2015

Abstracts

Hynek Baran (Oct. 18 – 24)

tba

Friday 23 Oct., 17^{45} (20 min.)

Diego Catalano Ferraioli (Oct. 18 – 24)

Nontrivial 1-parameter families of zero-curvature representations obtained via symmetry actions

Thursday 22 Oct., 18^{00} (40 min.)

We will discuss the problem of constructing a 1-parameter family α_λ of zero-curvature representations of an equation $\mathcal{E}$, by means of classical symmetry actions on a given zero-curvature representation α. By using the cohomology defined by the horizontal gauge differential of α, we provide an infinitesimal criterion which permits to identify all infinitesimal classical symmetries of $\mathcal{E}$ whose flow could be used to embed α into a nontrivial 1-parameter family α_λ of zero-curvature representations of $\mathcal{E}$. The results are illustrated with some examples.

Jan Cieśliński (Oct. 18 – 24)

Extended Lie point symmetries. The case of inhomogeneous NLS equation.

Thursday 22 Oct., 18^{45} (20 min.)

We present a notion of extended Lie point symmetries, i.e., symmetries of a family of differential equations, parameterized by one or more functions. Performing symmetry analysis we treat these functions as additional variables. Extended Lie point symmetries are computed as point symmetries with respect to all variables except those additional ones (they can transform into any function of the same type). One case, namely inhomogeneous Nonlinear Schrodinger equation, is of special interest, because applying this approach we obtain a group of nonlocal symmetries in an explicit form. The group parameter is related to the spectral parameter of the associated (nonisospectral) Lax pair. The main result is old but, in my opinion, a full interpretation of this phenomenon is still missing. Another challange is to find other interesting or useful examples. References: J. Cieslinski, J. Phys. A: Math. Gen. 26 (1993) L267; J. Cieslinski, J. Math. Phys. 34 (1993) 2372.

Felipe Contatto (Oct. 18 – 24)

Vortex-like solitons and Painlevé integrability

Friday 23 Oct., 9^{45} (30 min.)

The Abelian Higgs model admits topological solitons called vortices, which can be considered as finite energy solutions to the Taubes equation. They are static solutions on a two-dimensional Riemannian manifold. This equation is not integrable in general and few exact solutions are known (the most famous one being on hyperbolic surfaces). I will present a modified version of this model that generalises the usual theory and admits vortex-like solitons, then I will study integrability of the modified Taubes equation using Painlevé analysis and derive some exact solutions. Despite being defined on a smooth surface, these solitons have a secondary interpretation as vortices on a conifold. The approach illustrates how Painlevé analysis can be helpful in the construction of topological solitons.

Maciej Dunajski (Oct. 21 – 24)

First integrals of affine connections and Hamiltonian systems of hydrodynamic type

Friday 23 Oct., 9^{00} (40 min.)

We find necessary and sufficient conditions for a local geodesic flow of an affine connection on a surface to admit a linear first integral. We use this result to construct explicit obstructions to the existence of a Hamiltonian formulation of Dubrovin–Novikov type for a given one-dimensional system of hydrodynamic type. Joint with Felipe Contatto.

James M. Gundry (Oct. 18 – 24)

Higher symmetries of the Schrödinger operator

Friday 23 Oct., 10^{45} (20 min.)

Eastwood proved that the higher symmetry operators of the Laplacian on Euclidean space are given by conformal Killing tensors. I will describe a non-relativistic analogue of this result, in which the higher symmetry operators of the free-particle Schrödinger operator are identified with the "Schrödinger–Killing" tensors of a Newton–Cartan spacetime.

Gerard Helminck (Oct. 18 – 24)

Integrable deformations in matrix pseudo-differential operators

Monday 19 Oct., 11^{00} (40 min.)

We introduce various systems of compatible Lax equations in the matrix pseudo-differential operators, discuss their structure and show how one can obtain solutions of them.

Josef Janyška (Oct. 18 – 24)

On the Lie algebra of generators of infinitesimal symmetries almost-cosymplectic-contact structures

Friday 23 Oct., 16^{00} (40 min.)

An *almost-cosymplectic-contact (ACC) structure (pair)* on an odd dimensional manifold $\mathbf{M}$ is given by a pair (ω, Ω) of a 1-form and a 2-form such that

$$d\Omega = 0\,, \quad \omega \wedge \Omega^n \not\equiv 0\,.$$

As an infinitesimal symmetry of the ACC structure we assume a vector field X on $\mathbf{M}$ such that $L_X\omega = 0$ and $L_X\Omega = 0$. Infinitesimal symmetries are generated by pairs of functions (f, h) on $\mathbf{M}$. We show that generators of infinitesimal symmetries of (ω, Ω) form a Lie algebra. As an example we describe the Lie algebra of infinitesimal symmetries of the ACC structure of the phase space of the classical spacetime given as the 1-jet space of motions.

Igor Khavkine (Oct. 18 – 24)

Applications of compatibility complexes and their cohomology in relativity and gauge theories

Tuesday 20 Oct., 9^{00} (40 min.)

I will discuss the Killing operator ($K_{ab}[v] = \nabla_a v_b + \nabla_b v_a$) as an overdetermined differential operator and its (formal) compatibility complex. It has been recently observed that this compatibility complex and its cohomology play an important role in General Relativity. In more general gauge theories, an analogous role is played by the "gauge generator" operator and its compatibility complex. An important open problem is to explicitly compute the tensorial form of the compatibility complex on (pseudo-)Riemannian spaces of special interest. Surprisingly, despite its importance, the full compatibility complex is known in only very few cases. I have recently reviewed one of these cases, constant curvature spaces, where this complex is known as the Calabi complex, in [arXiv:1409.7212].

Jerzy Kijowski (Oct. 18 – 20)

Geometric quantization and Bäcklund transformations of the Schrödinger equations

Monday 19 Oct., 18^{00} (50 min.)

Theory of geometric quantization will be formulated in an original form. Unexpected symmetries of the Schrödinger equation will be derived this way. As a possible application, the new quantization method of the classical spin system will be presented.

Joseph Krasil'shchik (Oct. 18 – 24)

Nonlocal symmetries of the 3D rdDym equation

Thursday 22 Oct., 16^{00} (20 min.)

Using Lax representation with non-removable parameter, we construct two infinite hierarchies of nonlocal conservation laws for the 3D rdDym equation $u_{tu} = u_x u_{xy} - u_y u_{xx}$ and completely describe the algebras of nonlocal symmetries in the corresponding coverings. A joint work with H. Baran, O. Morozov and P. Vojčák.

Boris Kruglikov (Oct. 18 – 20)

Sub-maximal symmetry and integrability

Tuesday 20 Oct., 11^{00} (40 min.)

I will discuss the results on the gap problem in the symmetry dimensions of parabolic geometries (joint with D. The). Then I will tell about further progress for more general geometries (joint with V. Matveev, H. Winther, D. The), and relation to integrability in finite-type systems. I finish with speculations about the problem in the infinite-type case and infinite-dimensional Lie pseudogroups.

Lê Hông Vân (Oct. 19 – 22)

Homotopy invariants of chain complexes and closed orbits of locally Hamiltonian systems

Wednesday 21 Oct., 16^{00} (40 min.)

In 1987 Floer developed his Floer homology theory to prove a homological version of the Arnold conjecture, which is finally proved by Fukaya–Ono and Liu–Tian in 1998. In my lecture I shall discuss a homological version of the Arnold conjecture concerning the least number of closed orbits of locally Hamiltonian systems. I also discuss the strong version of the Arnold conjecture using homotopy invariants of free finitely generated chain complexes.

Paolo Lorenzoni (Oct. 18 – 24)

F-manifolds, multi-flat structures and Painlevé transcendents

Thursday 22 Oct., 9^{00} (40 min.)

We study F-manifolds equipped with multiple flat connections (and multiple F-products), that are required to be compatible in a suitable sense. In the first part of the talk we consider bi-flat F-manifolds and we show that in dimension three, they are locally parameterized by solutions of the full Painlevé IV,V and VI equations, according to the Jordan normal form of the operator of multiplication by the Euler vector field. In the second part of the talk we discuss conditions for the existence of multi-flat structures. In the semisimple case we show that a necessary condition can be expressed in terms of the integrability of a distribution of vector fields that are related to the unit vector fields for the multiple products involved. Using this fact we show that in general there can not be multi-flat structures with more than three flat connections. On the contrary, in the non-semisimple case, it is possible to construct multi-flat F-manifolds, with any number of compatible flat connections. Based on joint works with A. Arsie.

Valentin Lychagin (Oct. 18 – 23)

Finite dynamics and integrability for Rapoport–Leas models

Monday 19 Oct., 9^{45} (40 min.)

The Rapoport–Leas evolutionary equations describe a liquid displacement in porous medium. Finite-dimensional dynamics correspond to finite dimensional invariant submanifolds in a space of functions. Low dimensional dynamics as well as integrable models of the equations will be discussed during the talk.

Franco Magri (Oct. 18 – 22)

The Clebsch system

Monday 19 Oct., 9^{00} (40 min.)

In this talk I wish to illustrate a new method to bring to Abel form the equations of motion of a certain class of integrable Hamiltonian systems of interest in classical mechanics.

Giovanni Manno (Oct. 18 – 23)

Conformal geometric aspects of hyperplane sections of Lagrangian Grassmannians

Friday 23 Oct., 11^{10} (30 min.)

The Lagrangian Grassmannian $L(2,4)$ is a smooth 3-dimensional manifold naturally equipped with a conformal metric. We will use this structure to define a conformally invariant second-order differential operator whose vanishing characterizes the hyperplane sections of $L(2,4)$. We shall generalize such a result to $L(3,6)$,

where the natural conformal structure is no longer represented by a metric, but by a symmetric 3-tensor instead. This talk is based upon an ongoing work with G. Moreno and J. Gutt.

Luigi Martina (Oct. 18 – 24)

Structure-preserving discretizations of the Liouville equation

Wednesday 21 Oct., 18^{35} (30 min.)

Symmetry structures of partial differential equations can be reflected in difference schemes. In particular, the Liouville equation is a prototype of systems in which three different structure-preserving discretizations on four point lattices can be presented and, then, used to solve specific boundary value problems. The results are compared with exact solutions satisfying the same boundary conditions. One preserves linearizability of the equation, another the infinite-dimensional symmetry group as higher symmetries, the third preserves the maximal finite-dimensional subgroup of the symmetry group as point symmetries. A 9-point invariant scheme is also considered, but worse numerical solutions are presented and discussed.

Michal Marvan (Oct. 18 – 24)

On an integrable class of Chebyshev nets

Friday 23 Oct., 18^{10} (20 min.)

We study surfaces equipped with a Chebyshev net such that the Gauss curvature K and a curvature G of the net satisfy a linear condition $\alpha K+\beta G+\gamma=0$, where α,β,γ are constants. These surfaces form an integrable class. We point out some of its noteworthy peculiarities.

Oleg Morozov (Oct. 16 – 26)

Deformed cohomologies of symmetry pseudo-groups and coverings of differential equations

Tuesday 20 Oct., 17^{10} (20 min.)

I will talk about a relation between deformed cohomologies of symmetry pseudo-groups and coverings of differential equations. Examples will include the potential Khokhlov–Zabolotskaya equation and the Boyer–Finley equation.

Folkert Müller-Hoissen (Oct. 18 – 24)

Self-consistent sources for integrable equations via deformations of binary Darboux transformations

Monday 19 Oct., 16^{45} (40 min.)

We reveal the origin and structure of self-consistent source extensions of integrable equations from the perspective of binary Darboux transformations. They arise via a deformation of the potential that is central in this method. As examples, we obtain in particular matrix versions of self-consistent source extensions of the sine-Gordon, nonlinear Schrödinger, KP, Davey-Stewartson, two-dimensional Toda lattice and discrete KP systems. They are accompanied by a hetero binary Darboux transformation that generates solutions of such a system from a solution of the source-free system and solutions of an associated linear system and its adjoint. The essence of all this is encoded in universal equations in the framework of bidifferential calculus. This is joint work with Oleksandr Chvartatskyi and Aristophanes Dimakis.

Francesco Oliveri (Oct. 18 – 24)

Symmetries, equivalence and decoupling of first order PDEs

Thursday 22 Oct., 11^{00} (40 min.)

Systems of first order partial differential equations with the aim of identifying classes of equivalent ones (up to invertible point transformations) are considered. In particular, necessary and sufficient conditions are established in order to map a general nonautonomous and nonhomogeneous quasilinear first order system of partial differential equations to a quasilinear autonomous and homogeneous one. Also, the necessary conditions in order to map a nonlinear first order system to quasilinear autonomous and homogeneous form are given. Both results are obtained in the framework of Lie group analysis of differential equations, and some examples are provided. Finally, a general result concerned with the decoupling problem of a hyperbolic system of first order quasilinear partial differential equations is given.

Dominik Ostermayr (Oct. 18 – 24)

Harmonic maps from super Riemann surfaces into complex projective superspaces: Twistor lifts and integrable systems

Friday 23 Oct., 16^{45} (30 min.)

In their seminal paper, Eells and Wood classified isotropic harmonic maps from a Riemann surface into complex projective spaces via twistor lifts. This accounts for all harmonic maps if the source is a sphere. Moreover, Burstall showed that all non-isotropic harmonic 2-tori can be constructed by integrating commuting flows. I shall discuss the analogues of these two approaches for harmonic maps from a super Riemann surface into complex projective superspaces. In particular, I shall show how super Riemann surfaces with parabolic structures arise naturally in this context.

Andriy Panasyuk (Oct. 18 – 24)

Veronese webs and nonlinear PDEs

Wednesday 21 Oct., 18^{00} (30 min.)

It is known that a geomeric structure of a Veronese web is described by the Hirota dispersionless nonlinear equation. Seen as Lorentzian hyper-CR Einstein–Weyl space the same structure is given by the so-called hyper-CR equation. In this talk we propose a simple geometric procedure of generating different nonlinear PDEs describing Veronese webs and interpolating between two equations mentioned. Bäcklund transformations between different types will be also discussed. A joint work with Boris Kruglikov.

Maxim Pavlov (Oct. 18 – 24)

Gelfand–Dorfman Poisson brackets and classification of the corresponding three-dimensional quasilinear equations of the second order

Tuesday 20 Oct., 16^{25} (40 min.)

We construct Hamiltonian integrable hydrodynamic chains associated with infinitely many component Gelfand–Dorfman Poisson brackets. We describe the corresponding three-dimensional quasilinear equations of the second order.

Andrei Pogrebkov (Oct. 18 – 24)

Integrable discretizations of integrable PDE's

Monday 19 Oct., 16^{00} (40 min.)

We present a method of derivation of the linear differentiable equations that admit a "nonlinearization" to integrable differential equations. Developing an approach based on the commutator identities on associative algebras, we suggest a procedure of discretization of such nonlinear integrable equations that preserves the property of integrability.

Roman Popovych (Oct. 18 – 24)

Conservation laws of even-order evolution equation

Tuesday 20 Oct., 18^{00} (40 min.)

We solve the inverse problem on conservation laws of (1+1)-dimensional evolution equations. This allows us to exhaustively describe conservation laws of even-order evolution equations. In particular, we prove that if the dimension of the space of conservation laws of an even-order evolution equation is greater than its order, then this equation is linearizable by a contact transformation. As an illustrating example, we completely classify conservation laws of fourth-order evolution equations up to contact equivalence. Some related results are also discussed. A joint work with A. Sergyeyev.

Vladimir Roubtsov (Oct. 18 – 23)

Painlevé monodromy varieties and quantization

Thursday 22 Oct., 9^{45} (40 min.)

We discuss quantum algebras related to affine cubics arising as monodromy data varieties for confluented Painlevé equations.

We describe some examples of non-commutative cubics unifying the "quantum Painlevé cubics" and cubic superpotentials for 3D (Generalized) Sklyanin algebras. Such general potentials appear in a description of moduli spaces of vacuum states in $N = 4$ supersymmetric Yang–Mills field theory.

Alexey Samohin (Oct. 18 – 24)

Numeric simulation of sawtooth solutions of the Burgers equation on a finite interval

Tuesday 20 Oct., 18^{45} (20 min.)

Properties of the solutions to the Burgers equation $u_t = \varepsilon^2 u_{xx} - 2uu_x$ on a finite interval $x \in [0, L]$ are studied. The initial value/boundary conditions model a periodic perturbation on the left boundary:

$$u(x,0) = a, \quad u(0,t) = a + b\sin(\omega t), \quad u_x(L,t) = 0$$

The asymptotics of the solution for this problem at $L \to \infty$ coincides with the well known Fay solution

$$u = \frac{a}{R} \sum_{n=1}^{\infty} \frac{\sin(n\theta)}{\sinh(n(1+X)/2 \cdot R};$$

here R is the Reynolds number, $\theta = \omega(t - x/u_0)$. In particular, $\lim_{x\to+\infty} u(x,t) = a$, which is the solution's average value over $x > 0$. Not so for another asymptotics, at $t \to +\infty$. The form of the solution retains the sawtooth profile yet its average over $[0, L]$ differs from a and depends also on the perturbation amplitude b. Interaction between two perturbations of different frequencies is discussed.

Artur Sergyeyev (Oct. 18 – 24)

TBA

Thursday 22 Oct., 16^{25} (40 min.)

Luca Vitagliano (Oct. 18 – 24)

Dirac–Jacobi bundles

Wednesday 21 Oct., 16^{45} (40 min.)

Dirac structures encompass both presymplectic and Poisson structures and represent a natural arena where studying Hamiltonian systems with both symmetries and constraints. I will present a contact/Jacobi analogue of Dirac structures, named Dirac-Jacobi structures, and their main properties.

Raffaele Vitolo (Oct. 18 – 24)

On the Hamiltonian structure of hydrodynamic type systems in conservative form

Tuesday 20 Oct., 16^{00} (20 min.)

We find a new criterion which allows us to effectively reconstruct third-order homogeneous Hamiltonian operators for a hydrodynamic-type system which is written as a system of conservation laws. Conversely, the criterion can be used to describe all possible hydrodynamic type systems for each given third order homogeneous Hamiltonian operator. We apply the solution of the first problem to finding third-order homogeneous Hamiltonian operators for WDVV systems. We solve the second problem for the 3-component case using our classification of 3-component homogeneous third-order Hamiltonian operators.

Petr Vojčák (Oct. 18 – 24)

Some integrability properties of a $(3+1)$-dimensional integrable generalization of the dKP equation

Thursday 22 Oct., 17^{10} (20 min.)

We present some integrabitity properties of the following system of PDE's

$$\begin{aligned} q_z &= 2u_z + w_x + 2ww_z, \\ v_z &= 2q_x - 3u_x - 2w_y + 2wu_z - 2ww_x + 2uw_z, \\ u_t &= vu_z + qu_x - uv_z - wv_x + v_y, \\ w_t &= q_y - 2v_x + 4wu_x - wq_x + qw_x + vw_z - uq_z, \end{aligned}$$

which provides a (3+1)-dimensional integrable generalization of the dispersionless Kadomtsev–Petviashvili (dKP) equation [see A. Sergyeyev, A new class of (3+1)-dimensional integrable systems related to contact geometry, arXiv:1401.2122v2]. A joint work with H. Baran and A. Sergyeyev.

Pasha Zusmanovich (Oct. 18 – 24)

Invariants of Lie algebras generated by dynamical systems

Tuesday 20 Oct., 9^{45} (40 min.)

At the beginning of 1990s, Vershik introduced an interesting class of Lie algebras which he dubbed as "Lie algebras generated by dynamical systems." We will describe an approach for computations of central extensions, invariant symmetric bilinear forms, and other invariants of these algebras, useful for applications in integrability of dynamical systems. The approach is inspired by a similar approach to such questions for another class of algebras – Lie algebras represented as a tensor product of algebras over Koszul dual operads.