Can gravity be repulsive?

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Outline

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Vacuum Einstein metrics with 2-dimensional Killing leaves. I Local aspects, Differential Geometry and its Applications (2002) 16(2)95-120

Vacuum Einstein metrics with 2-dimensional Killing leaves. II Global aspects, Differential Geometry and its Applications (2002) 17(1)15-35

Outline


Differential Geometry
Quantum Field Theory
Repulsive Gravity
General Relativity
Gravitational waves

Outline

Einstein Field Equations

$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} + \Lambda \, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Special properties of some wave-like exact solutions

Outline


End-station @ 4 km
Mid-station @ 2 km
4 km (2.5 mi.)

- **Linearization**
- $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $|h_{\mu\nu}| \ll 1$ and $|\partial_\alpha h_{\mu\nu}| \ll 1$
- $\eta^{\alpha\beta}\partial_{\alpha\beta}h_{\mu\nu} = 0$ (vacuum), with $\eta^{\alpha\mu}[\partial_\alpha(h_{\mu\nu}) - \frac{1}{2}\partial_\nu(h_{\alpha\mu})] = 0$,
- $\eta^{\alpha\beta}\partial_{\alpha\beta}h_{\mu\nu} = -16\pi G c^{-4}(T_{\mu\nu} + \tau_{\mu\nu})$, $\eta^{\alpha\mu}[\partial_\alpha(h_{\mu\nu}) - \frac{1}{2}\partial_\nu(h_{\alpha\mu})] = 0$
- $h = \eta^{\rho\sigma}h_{\rho\sigma}$
- $R^{(1)}_{\mu\nu} = \eta^{\alpha\beta}\partial_{\alpha\beta}h_{\mu\nu}$

Waves

A wave in the sense of Hadamard is a field which is regularly discontinuous across a moving surface, which is called the wave-front. However, the name wave is commonly given to the wave-front, rather than to the field.

In any case, waves in the sense of Hadamard correspond to the propagation of discontinuities of physical quantities describing either fields (essentially electromagnetic and gravitational field) or the motion of a fluid. In this framework, an ordinary gravitational wave is a discontinuity hypersurface for the Riemann curvature tensor:

$$[R_{\alpha,\beta,\gamma,\rho}] \neq 0$$

Let Ω be an open domain of space-time $\mathcal{M}$ (with compact closure) and Σ be a hypersurface in Ω. Let the second derivatives of g be regularly discontinuous (eventually with null discontinuity) across Σ. Let us denote by $[\Phi]$ the jump across Σ of a generic regularly discontinuous function Φ. Let $f(x) = 0$ be the equation of Σ and $l_\alpha = \partial_\alpha f$

The metric discontinuity $\partial^2 g_{ab}$ is a well defined field on Σ, such that the following Hadamard compatibility conditions hold

$$[\partial_\alpha \partial_\beta \; g_{\rho\sigma}] = l_\alpha l_\beta \partial^2 g_{\rho\sigma}$$

We have:

$$2[R_{\alpha\beta\rho\sigma}] = l_\beta l_\rho \partial^2 g_{\alpha\sigma} - l_\beta l_\sigma \partial^2 g_{\alpha\rho} - l_\alpha l_\rho \partial^2 g_{\beta\sigma} + l_\alpha l_\sigma \partial^2 g_{\beta\rho}$$

Outline

- The interest in repulsive gravity, or *antigravity* as it was usually called, goes back to the fifty's (Morrison and Gold '58, Morrison '58, Nieto and Goldman '91)

The general point of view was that since gravitational interaction is mediated by a spin-2 particle, it can only be attractive and thus, to obtain a repulsive behavior, some other ingredient is required. The idea was then to explore the possibility of repulsive matter-antimatter gravity, but within the old quantum field theories there was no room for such a possibility.

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- The main arguments, reviewed in (Nieto and Goldman '91), were of various kinds including violation of energy conservation and disagreement with experiments of the Eötvös type due to the effects of antigravity on the vacuum polarization diagrams of atoms.

  More recently however, within the context of modern quantum field theories, it was proven that those arguments were no longer sufficient to exclude repulsive effects and the interest in antigravity increased again. For example, in 1992 Fabbrichesi and Roland shown that in supergravity and string theory, due to dimensional reduction, the effective 4-dimensional theory of gravity may show repulsive aspects because of the appearance of spin-1 graviphotons.

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- Repulsive behavior of gravitational interaction, in particle physics, can be associated to specific properties of some exact solutions of Einstein Equations. Our point of view is the following. In the usual treatment of gravitational waves only Fourier expandable solutions of d'Alembert equation are considered; then it is possible to choose a special gauge (TT-gauge) which kills the spin-0 and spin-1 components.

  However there exist (see section 2 and 3) physically meaningful solutions (Peres 1959 Stephani 1996, Canfora, Vilasi and Vitale 2002 Stephani, Kramer, MacCallum, Honselaers and Herlt 2003,) of Einstein equations which are not Fourier expandable and nevertheless whose associated energy is finite.

- For some of these solutions the standard analysis shows that spin-1 components cannot be killed (Canfora and Vilasi 2004, Canfora, Vilasi and Vitale 2004); this implies that repulsive aspects of gravity are possible within pure General Relativity, i.e. without involving spurious modifications. In previous works it was shown that light is among possible sources of such spin-1 waves (Vilasi 2007).

Outline

- **The harmonic gauge**
  A gravitational field $g = g_{\mu\nu}(x)\, dx^{\mu} dx^{\nu}$ is said to be *locally weak* if there exists a (harmonic) coordinates system and a region $M' \subset M$ of space-time M in which the following conditions hold:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| << 1, \qquad |h_{\mu\nu,\alpha}| << 1. \tag{1}$$

In the weak field approximations and in harmonic coordinates system, Einstein field equations reduce to the wave equation for $h_{\mu\nu}$. The choice of the harmonic gauge plays a key role in this reduction; no other special assumption, either on the form or on the analytic properties of the perturbation h, is necessary.

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For globally *square integrable* solutions of the wave-equation (that is, solutions which are square integrable on the whole of M), with a suitable gauge transformation preserving the harmonicity of the coordinate system and the "weak character" of the field, one can always kill the “spin-0” and ”spin-1” components of the gravitational waves. However, in the following we will meet some interesting solutions which do not belong to this class.

- **Gravitodynamics**

  A slightly different point of view, which is useful in clarifying the nature of spin of gravitational waves is provided by the *gravitodynamics*, henceforth GD(see, for example, Mashoon 2008). In this scheme one tries to exploit as much as possible the similarities between the Maxwell and the linearized Einstein equations. To make this analogy evident it is enough to write a weak gravitational field fulfilling conditions in the GD form.

$$ds^2 = (-1 + 2\Phi^{(g)})dt^2 - 4(\mathbf{A}^{(g)} \cdot d\mathbf{x})dt + (1 + 2\Phi^{(g)})\delta_{ij}dx^i dx^j, \quad (2)$$

with

$$h_{00} = 2\Phi^{(g)}, \qquad h_{0i} = -2A_i^{(g)}.$$

- **Gravito-Lorentz gauge**

  Hereafter the spatial part of 4-vectors will be denoted in bold and the standard symbols of 3-dimensional vector calculus will be adopted. In terms of $\Phi^{(g)}$ and $\mathbf{A}^{(g)}$ the harmonic gauge condition reads

$$\frac{\partial \Phi^{(g)}}{\partial t} + \frac{1}{2}\nabla \cdot \mathbf{A}^{(g)} = 0, \qquad (3)$$

  and, once the gravitoelectric and gravitomagnetic fields are defined in terms of GD potentials, as

$$\mathbf{E}^{(\mathbf{g})} = -\nabla\Phi^{(g)} - \frac{1}{2}\frac{\partial \mathbf{A}^{(g)}}{\partial t}, \qquad \mathbf{B}^{(\mathbf{g})} = \nabla \wedge \mathbf{A}^{(g)},$$

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- **Gravito-Faraday tensor**

  Gravitational waves can be also described in analogy with electromagnetic waves, the gravitoelectric and the gravitomagnetic components of the metric being

$$E_\mu^{(g)} = F_{\mu 0}^{(g)}; \qquad B^{(g)\mu} = -\varepsilon^{\mu 0 \alpha\beta} F_{\alpha\beta}^{(g)}/2 \quad ,$$

  where

$$\begin{aligned} F_{\mu\nu}^{(g)} &= \partial_\mu A_\nu^{(g)} - \partial_\nu A_\mu^{(g)} \\ A_\mu^{(g)} &= -h_{0\mu}/2 = (-\Phi^{(g)}, \mathbf{A}^{(g)}). \end{aligned}$$

- **Geodesic motion**

  The first order geodesic motion for a *massive particle* moving with velocity $v^{\mu} = (1, \underline{\mathrm{v}}), |\mathbf{v}| << \mathbf{1}$, in a light beam gravitational field characterized by gravitoelectric $\mathbf{E}^{(g)}$ and gravitomagnetic $\mathbf{B}^{(g)}$ fields, is described (at first order in $|\mathbf{v}|$) by the *acceleration*:

$$\mathbf{a}^{(g)} = -\mathbf{E}^{(g)} - 2\mathbf{v} \wedge \mathbf{B}^{(g)}.$$

Outline

- **The photon gravitational interaction**

  The photon-photon and photon-electron scatterings may occur through the creation and annihilation of virtual electron-positron pairs and may even lead to collective photon phenomena. Photons also interact gravitationally but the gravitational scattering of light by light has been much less studied.

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They also discovered that null rays behave differently according to whether they propagate parallel or antiparallel to a steady, long, straight beam of light, but they didn't provide a physical explanation of this fact.

Results of Tolman, Ehrenfest, Podolsky, Wheeler were clarified in part by Faraoni and Dumse (1999), in the setting of classical pure General Relativity, the general point of view being that gravitational interaction is mediated by a spin-2 particle.

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Outline

Geometric properties

In previous papers (G. Sparano, G. Vilasi, G. Vinogradov (2001-2002), F. Canfora, G. Vilasi, P. Vitale 2002-2004) a family of exact solutions, namely g, of Einstein field equations, representing the gravitational wave generated by a beam of light, has been explicitly written

$$g = 2f(dx^2 + dy^2) + \mu \left[(w(x, y) - 2q)dp^2 + 2dpdq \right], \tag{4}$$

where $\mu(x, y) = A\Phi(x, y) + B$ (with $\Phi(x, y)$ a harmonic function and A, B numerical constants), $f(x, y) = (\nabla\Phi)^2 \sqrt{|\mu|}/\mu$, and $w(x, y)$ is solution of the *Euler-Darboux-Poisson equation:*

$$\Delta w + (\partial_x \ln |\mu|)\, \partial_x w + (\partial_y \ln |\mu|)\, \partial_y w = \rho,$$

$T_{\mu\nu} = \rho\delta_{\mu 3}\delta_{\nu 3}$ representing the energy-momentum tensor and Δ the Laplace operator in the (x, y) −plane.

Previous metric is invariant for the non Abelian Lie agebra $\mathcal{G}_2$ of Killing fields generating a 2-dimensional distribution $\mathcal{D}$, say

$$X = \frac{\partial}{\partial p}, \qquad Y = \exp\left(p\right)\frac{\partial}{\partial q},$$

with $[X, Y] = Y$, $g\left(Y, Y\right) = 0$ *and whose orthogonal distribution* $\mathcal{D}^{\perp}$ *is integrable.*

A full classification of metric tensor fields on a 4-dimensional manifold which are invariant for a Lie algebra of vector fields generating a 2-dimensional distribution $\mathcal{D}$, say X and Y with [X,Y] =sY ($s = 0, 1$), is fully described in Sparano, Vilasi, Vinogradov (2001-2002); there the Lie algebra is denoted with $\mathcal{A}_2$ in the Abelian case ($s = 0$) and with $\mathcal{G}_2$ in the non Abelian case ($s = 1$). Denoting with $\mathcal{D}^{\perp}$ the distribution orthogonal to $\mathcal{D}$ and with r the rank of metric when restricted to the leaves of $\mathcal{D}$, all possible cases are exhaustively described by the table below where the cases completely solved are indicated with bold face characters.

Table: 2−dimensional Lie Algebra metric classification

	$\mathcal{D}^\perp$, $r=0$	$\mathcal{D}^\perp$, $r=1$	$\mathcal{D}^\perp$, $r=2$
$\mathcal{G}_2$	**integrable**	**integrable**	**integrable**
$\mathcal{G}_2$	**semi-integrable**	**semi-integrable**	**semi-integrable**
$\mathcal{G}_2$	**non-integrable**	**non-integrable**	non-integrable
$\mathcal{A}_2$	**integrable**	**integrable**	integrable
$\mathcal{A}_2$	semi-integrable	semi-integrable	semi-integrable
$\mathcal{A}_2$	**non-integrable**	**non-integrable**	non-integrable

In the particular case $s = 1$, $f = 1/2$ and $\mu = 1$, the above family is locally diffeomorphic to a subclass of Peres solutions and, by using the transformation

$$p = \ln |u| \qquad q = uv,$$

can be written in the form

$$g = dx^2 + dy^2 + 2dudv + \frac{w}{u^2} du^2, \tag{5}$$

with $\Delta w(x, y) = \rho$, and has the Lorentz invariant *Kerr-Schild* form:

$$g_{\mu\nu} = \eta_{\mu\nu} + V k_\mu k_\nu, \qquad k_\mu k^\mu = 0.$$

- **Wave Character**
  The wave character and the polarization of these gravitational fields has been analyzed in many ways. For example, the Zel'manov criterion (see Zakharov 1973) was used to show that these are gravitational waves and the propagation direction was determined by using the Landau-Lifshitz pseudo-tensor. However, the algebraic Pirani criterion is easier to handle since it determines both the wave character of the solutions and the propagation direction at once. Moreover, it has been shown that, in the vacuum case, the two methods agree. To use this criterion, the Weyl scalars must be evaluated according to Petrov classification(Petrov 1969).

In the Newmann-Penrose formulation (Penrose 60) of Petrov classification, we need a *tetrad* basis with two real null vector fields and two real spacelike (or two complex null) vector fields. Then, if the metric belongs to type **N** of the Petrov classification, it is a gravitational wave propagating along one of the two real null vector fields (Pirani criterion). Let us observe that ∂_x and ∂_y are spacelike real vector fields and $\partial_v$is a null real vector but ∂_u is not. With the transformation $x \mapsto x, \quad y \mapsto y, \quad u \mapsto u, \quad v \mapsto v + w(x,y)/2u$, whose Jacobian is equal to one, the metric (5) becomes:

$$g = dx^2 + dy^2 + 2dudv + dw(x,y)dln|u|. \tag{6}$$

Since ∂_x and ∂_y are spacelike real vector fields and $\partial_u$and ∂_v are null real vector fields, the above set of coordinates is the right one to apply for the Pirani's criterion.

Since the only nonvanishing components of the Riemann tensor, corresponding to the metric (6), are

$$R_{iuju} = \frac{2}{u^3}\partial^2_{ij} w(x,y), \qquad i,j = x,y$$

these gravitational fields belong to Petrov type **N**(Zakharov 73). Then, according to the Pirani's criterion, previous metric does indeed represent a gravitational wave propagating along the null vector field ∂_u.

It is well known that linearized gravitational waves can be characterized entirely in terms of the linearized and gauge invariant Weyl scalars. The non vanishing Weyl scalar of a typical spin-2 gravitational wave is Ψ_4. Metrics (6) also have as non vanishing Weyl scalar Ψ_4.

- **Spin**
  Besides being an exact solution of Einstein equations, the metric (6) is, for $w/u^2 << 1$, also a solution of linearized Einstein equations, thus representing a perturbation of Minkowski metric $\eta = dx^2 + dy^2 + 2dudv = dx^2 + dy^2 + dz^2 - dt^2$ (with $u = (z-t)/\sqrt{2} \quad v = (z+t)/\sqrt{2}$) with the perturbation, generated by a light beam or by a photon wave packet moving along the z-axis, given by

$$h = dw(x,y)dln|z-t|,$$

  whose non vanishing components are

$$h_{0,1} = -h_{13} = -\frac{w_x}{(z-t)} \quad h_{0,2} = -h_{23} = -\frac{w_y}{(z-t)}$$

- A transparent method to determine the spin of a gravitational wave is to look at its physical degrees of freedom, *i.e.* the components which contribute to the energy (Dirac 75). One should use the Landau-Lifshitz (pseudo)-tensor t^{μ}_{ν} which, in the asymptotically flat case, agrees with the Bondi flux at infinity (F.Canfora, G.Vilasi and P. Vitale 2004). It is worth to remark that the canonical and the Landau-Lifchitz energy-momentum pseudo-tensors are true tensors for Lorentz transformations. Thus, any Lorentz transformation will preserve the form of these tensors; this allows to perform the analysis according to the Dirac procedure. A globally square integrable solution $h_{\mu\nu}$ of the wave equation is a function of $r = k_{\mu}x^{\mu}$ with $k_{\mu}k^{\mu} = 0$.

- With the choice $k_\mu = (1, 0, 0, -1)$, we get for the energy density t_0^0 and the energy momentum t_0^3 the following result:

$$16\pi t_0^0 = \frac{1}{4}\left(u_{11} - u_{22}\right)^2 + u_{12}^2, \qquad t_0^0 = t_0^3$$

where $u_{\mu\nu} \equiv dh_{\mu\nu}/dr$. Thus, the physical components which contribute to the energy density are $h_{11} - h_{22}$ and h_{12}. Following the analysis of Dirac 1975, we see that they are eigenvectors of the infinitesimal rotation generator $\mathcal{R}$, in the plane $x - y$, belonging to the eigenvalues $\pm 2i$. The components of $h_{\mu\nu}$ which contribute to the energy thus correspond to spin-2.

- In the case of the prototype of spin-1 gravitational waves (6), both Landau-Lifchitz energy-momentum pseudo-tensor and Bel-Robinson tensor (1958) single out the same wave components and we have:

$$\tau_0^0 \sim c_1(h_{0x,x})^2 + c_2(h_{0y,x})^2, \quad t_0^0 = t_0^3$$

where c_1 e c_2 constants, so that the physical components of the metric are h_{0x} and h_{0y}. Following the previous analysis one can see that these two components are eigenvectors of $i\mathcal{R}$ belonging to the eigenvalues ± 1. In other words, metrics (6), which are not pure gauge since the Riemann tensor is not vanishing, represent spin-1 gravitational waves propagating along the $z-$axis at light velocity.

- **Summarizing**
  Globally square integrable spin-1 gravitational waves propagating on a flat background are always pure gauge.
- Spin-1 *gravitational waves which are not globally square integrable are not pure gauge*. It is always possible to write metric (6) in an apparently transverse gauge (Stefani 96); however since these coordinates are no more harmonic this transformation is not compatible with the linearization procedure.
- What truly distinguishes spin-1 from spin-2 gravitational waves is the fact that in the spin-1 case the Weyl scalar has a non trivial dependence on the transverse coordinates (x, y) due to the presence of the harmonic function. This could led to observable effects on length scales larger than the *characteristic length scale* where the harmonic function changes significantly.

- Indeed, the Weyl scalar enters in the geodesic deviation equation implying a non standard deformation of a ring of test particles breaking the invariance under of π rotation around the propagation direction. Eventually, one can say that there should be distinguishable effects of spin-1 waves at suitably large length scales.
- It is also worth to stress that the results of Aichelburg and Sexl 1971, Felber 2008and 2010, van Holten 2008 suggest that the sources of asymptotically flat *pp*-waves (which have been interpreted as spin-1 gravitational waves (Canfora, Vilasi and Vitale 2002 and 2004) repel each other. Thus, in a field theoretical perspective (see Appendix), *pp*-gravitons must have spin-1 .

Outline

In the previous strong field regime, geodesic motion can be still described by using the GD formalism and we have

$$\mathbf{E}^{(g)} = -\frac{1}{2}(w_x, w_y, \frac{w}{u})u^{-2},$$

$$\mathbf{B}^{(g)} = \frac{1}{2}(w_y, -w_x, \frac{w}{u})u^{-2}.$$

Thus, "gravitational acceleration" acting over a massless particle is given by

$$\mathbf{a}^{(g)} = -[w_x(1-v_z)\mathbf{i} + w_y(1-v_z)\mathbf{j} + (w_x v_x + w_y v_y)\mathbf{k}]/2u^2. \qquad (7)$$

Rather than geodesic orbits, the motion of spinning particles, should be described by Papapetrou equations

$$\frac{D}{D\tau}(mv^{\alpha} + v_{\sigma}\frac{DS^{\alpha\sigma}}{D\tau}) + \frac{1}{2}R^{\alpha}_{\sigma\mu\nu}v^{\sigma}S^{\mu\nu} = 0,$$

where $S^{\mu\nu}$ is the *angular momentum tensor* of the spinning particle and

$$S^{\alpha} = \frac{1}{2}\epsilon^{\alpha\beta\rho\sigma}v_{\beta}S_{\rho\sigma}$$

defines the *spin four-vector* of the particle. These equations have been extended to the case of massless spinning particles by Mashhoon (Ma75, BCGJ06).

However, assuming that the spin is directed along the z-axis $\mathbf{S} = (0, 0, S_z)$, Papapetrou equations for photons coincide, in the gravitational field represented by Eq.(6), with usual geodesic equations and no additional contributions must be added to the "gravitational acceleration" $\mathbf{a}^{(g)}$ given by Eq.(7). The velocity $\mathbf{v}$ of a photon is determined by the null geodesics equations

$$(h-1) - 2hv_z + (h+1)v_z^2 = 0$$

which has two solutions

$$v_z = 1, \qquad v_z = \frac{h-1}{h+1} = \frac{w-u^2}{w+u^2}$$

.

If photon propagates parallel to the light beam (lb), $v = (0, 0, 1)$, then $\mathbf{a}^{(g)} = 0$ and there is no attraction or repulsion.
If the photon propagates antiparallel to lb, $v = (h-1)/(h+1)$, then the acceleration is not vanishing

$$\mathbf{a}^{(g)} = -\nabla w / 2 \left(w + u^2 \right)$$

and photons attract each other.

Thus, the lack of attraction found by Tolman, Ehrenfest, Podolsky (later also analysed by Wheeler, Faraoni and Dumse) comes out also from the analysis of the geodesical motion of a massless spin-1 test particle in the strong gravitational field of the light, neglecting however the gravitational field generated by that particle.

An exhaustive answer could derive only determining the gravitational field generated by two photons, each one generating spin-1 gravitational waves. However, since helicity seems to play for photons the same role that charge plays for charged particles, two photons with the same helicity should repel one another.

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This repulsion turns out to be very weak and cannot be certainly observed in laboratory but it could play a relevant role at cosmic scale and could give not trivial contributions to the dark energy.

Therefore, together with gravitons (spin-2), one may postulate the existence of graviphotons (spin-1) and graviscalar (spin-0). Through coupling to fermions, they might give forces depending on the barion number.

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- It is known from Quantum Field Theory (QFT) that a consequence of spin-1 messengers is that particles with the same orientation repel and particles with opposite orientation attract. Indeed, the path integral formalism describing a massive vector field theory A_μ makes use of the *partition function* which can be represented by using the Feynman path integral

$$Z(J) = \int DA \exp[\frac{i}{\hbar} S(A, J)],$$

where

$$S(A, J) = \int d^4x \left(A_\mu \left[\left(\partial^2 + m^2\right) g^{\mu\nu} - \partial^\mu \partial^\nu\right] A_\nu + J^\mu A_\mu\right)$$

is the classical action.

- We also have (see for instance (Zee 2003)

$$Z(J) = \exp[\frac{i}{\hbar} W(J)],$$

with

$$W(J) = -\frac{1}{2}\int d^4x d^4y J^{\mu}(x) D_{\mu\nu}(x-y) J^{\nu}(y),$$

where $D_{\nu\lambda}(x)$ is the Green function defined by

$$\left[\left(\partial^2 + m^2\right) g^{\mu\nu} - \partial^{\mu}\partial^{\nu}\right] D_{\nu\lambda}(x) = \delta^{\mu}_{\lambda}\delta^{(4)}(x).$$

Taking the Fourier transform, we get

$$W\left(J\right)=-\frac{1}{2}\int d^{4}kJ^{\mu*}\left(k\right)D_{\mu\nu}\left(k\right)J^{\nu}\left(k\right),$$

where

$$D_{\mu\nu}\left(k\right)=\frac{-g_{\mu\nu}+k_{\mu}k_{\nu}/m^{2}}{k^{2}-m^{2}}$$

is called the *propagator for the massive vector field* A_{μ}. A simple calculation of the potential energy between like charges gives

$$U\backsim\frac{\exp\left(-mr\right)}{4\pi r},$$

so that $dU/dr<0$ and the force between like charges turns out to be repulsive, as we already know from electrodynamics.

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