Local and Nonlocal Geometry of PDEs and Integrability

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Hexagonal geodesic 3-webs

Sergey Agafonov
São José do Rio Preto, Brazil

Abstract: We prove that a surface carries a hexagonal 3-web of geodesics if and only if the geodesic flow on the surface admits a cubic first integral and show that the system of partial differential equations, governing metrics on such surfaces, is integrable by generalized hodograph transform method.

We present some new local examples of such metrics, discuss known ones, and establish an analogue of the celebrated Graf and Sauer Theorem for Darboux superintegrable metrics.

Shortest and straightest geodesics of an invariant sub-Riemanniasn metric on a flag manifold

Dmitri Alekseevsky
Moscow, Russia

Abstract: There are different equivalent definitions of geodesics of a Riemannian manifold (M, g):

- According to Euler-Lagrange variational definition, geodesics are extremals of length functional or functional of kinetic energy that is shortest curves, joint two closed points.
- According to Hamilton approach, geodesics are projection to M of integral curves of the Hamiltoian flow on (T^*M, ω_{can}) with the quadratic Hamiltonian $H(p) = \frac{1}{2}g^{-1}(p, p)$, $p \in T^*M$.
- According to Levi-Civita, geodesics are straightest curves, i.e. curves $\gamma(s)$ whose tangent field $\dot{\gamma}(s)$ is parallel with respect to Levi-Civita connection.

These definitions may be generalized to the sub-Riemannian manifold (M, D, g) where g is a sub-Riemannian metric, defined on a bracket generated distribution D. However , as it was remarked by A.M. Vershik and L.D. Faddeev, variational definition of sub-Riemannian geodesics as "shortest" horisontal curves (used in control theory and many applications) is different from definition of geodesics as straightest curves , used in non-holonomic mechanics. Moreover, they proved that generically shortest geodesics are different from straightest geodesics and indicate examples when these to notions are equivalent.

We recall a Schouten-Wagner description of straightest sub-Riemannian geodesics as geodesics of a partial connection and definition of Wagner curvature tensor of such connection.

Then we classify invariant sub-Riemannian structures (D, g) on flag manifolds (i.e. adjoint orbits of a compact simple Lie group) and study relations between shortest and strainghest sub-Riemannian geodesics of such sub-Riemannian homogeneous manifold.

Differential invariants and representations of semisimple algebraic groups

Pavel Bibikov
Moscow, Russia

Abstract: In this talk we discuss an approach to the study of orbits of actions of semisimple algebraic groups in their irreducible complex representations, which is based on differential invariants on the one hand, and on geometry of reductive homogeneous spaces on the other hand.

We will start from the well-known problem of SL_2-classification of binary forms, which was studied by many famous mathematicians during XIX and XX centuries. Classical invariant theory starts from this problem. But it appears that the full solution of this problem can be obtained with the help of differential equations and differential invariants. Namely, we represent each binary form of degree n as a solution of the so-called Euler equation $xu_x + yu_y = nu$, and study differential invariants for the SL_2-actions on the prolongations of this equation. We prove that the dependence between basic differential invariant and its derivations uniquely defines the SL_2-orbit of a given binary form. We also present some examples.

It the second part of the talk we generalize this approach for the problem of classification of G-orbits of a given connected semisimple algebraic group G in its irreducible representation. According to the Borel-Weil-Bott theorem, every irreducible representation of connected semisimple Lie group is isomorphic to the action of this group on the module of holomorphic sections of some one-dimensional bundle over the flag variety G/B. Using this, we give a complete description of the structure of the field of differential invariants for this action and obtain a criterion which separates regular G-orbits. Also we discuss the applications of these results to the classical invariant theory.

In collaboration with Valentin Lychagin.

Cohomology of Lie algebroids on schemes and applications

Ugo Bruzzo
Trieste, Italy

Abstract: I will consider Lie algebroids on noetherian separated schemes and will show how their cohomology can be described as a derived functor. I will also describe applications to the nonabelian extension problem for such Lie algebroids. (Partly in collaboration with E. Aldrovandi and V. Rubtsov).

A Darboux-Getzler theorem for scalar difference Hamiltonian operators

Matteo Casati
Canterbury, UK

Abstract: The classification of Hamiltonian operators in the formal calculus of variations relies on their corresponding Poisson-Lichnerowicz cohomology. We consider the case of scalar difference Hamiltonian operators, such as the ones which constitute the biHamiltonian pair for the Volterra chain, and prove that 1) the normal form of a order 1 scalar difference operator P is constant; 2) $H^p(P) = 0\ \forall p > 1$, so that in particular there are not nontrivial infinitesimal deformations and any infinitesimal deformation can be extended to an Hamiltonian operator; 3) as a consequence, any higher order compatible Hamiltonian operator can be brought to the constant, order 1 form by a (Miura-like) change of coordinates.

Invertible linear ordinary differential operators and their generalizations

Vladimir Chetverikov
Moscow, Russia

Abstract: We consider invertible linear ordinary differential operators whose inversions are also differential operators. To each such operator one assigns a numerical table. We describe these tables in the elementary geometrical language. The table does not uniquely determine the operator. We present mathematical structures that should be specified for its unique determination.

We say that a linear differential operator is unicellular, if in some bases of the modules the operator is given by an upper triangular matrix that differs from the identity matrix only by the first row. The numerical tables of unicellular operators are of the simplest form. We show that any invertible linear ordinary differential operator is represented as compositions of unicellular ones.

These results are generalized to invertible mappings of filtered modules generated by one differentiation. Invertible linear ordinary differential operators, invertible linear difference operators with periodic coefficients, unimodular matrices, and C-transformations of control systems determine mappings of this type. The possibility of generalization of these results to partial differential operators is also discussed.

Solitons on Wormholes

Maciej Dunajski
Cambridge, UK

Abstract: The sine-Gordon equation in 1+1 dimensions admits a static kink solution with with topological charge 1. The kinks do not exist (for neither SG nor any other scalar field theory) in flat $d+1$ dimensions, where $d > 1$. I shall prove the existence of static kinks on $3+1$ dimensional curved wormhole space-time with two asymptotically flat regions, and discuss linear and non-linear stability of the kinks.

Approximate Conditional Symmetries of PDEs

Matteo Gorgone
Messina, Italy

Abstract: Following a recently introduced approach to approximate Lie symmetries which is consistent with the principles of perturbative analysis of differential equations containing small terms, the case of approximate Q-conditional symmetries is considered. An application of the method to a hyperbolic variant of a nonlinear reaction-diffusion-convection equation is exploited. Some approximate solutions are explicitly constructed.

Gauge PDE and AKSZ sigma models

Maxim Grigoriev
Moscow, Russia

Abstract: AKSZ sigma models were originally proposed to describe topological systems. In so doing the Batalin-Vilkovisky formulation of the system is encoded in the pair (target space and the source) of Q-manifolds. In fact, an AKSZ model with finite-dimensional target and space-time dimension higher than 1 is necessarily topological. It turns out that generic gauge PDE can be cast into the AKSZ sigma model whose target space is the BRST-extended equation manifold equipped with the BRST differential and the horizontal differential. This gives a rather flexible and powerful framework to study gauge PDE. I plan to discuss various applications of the framework including invariant gauge PDE, higher spin gauge theories and their holographic relations.

A construction of solutions of the strict n-component KP-hierarchy

Gerard Helminck
Amsterdam, Netherlands

Abstract: After a description of the strict n-component KP-hierarchy and its linearization, I will show how to construct geometrically solutions of this integrable hierarchy.

Complex invariant Einstein metrics on flag manifolds with T-root system BC_2

Aleksei Ivanov
Moscow, Russia

Abstract: Complex G-invariant metrics on flag manifold $M = G/H$ are parameterised by algebraic torus $(\mathbb{C}^*)^n$ and corresponding Einstein equations have form of Laurent polynomial equations. So we can consider Newton polytope P of Einstein system which depends only on T-root system of manifold M. Hence, according to Bernstein-Kushnirenko theorem, the number $E(M)$ of isolated complex invariant Einstein metrics (up to multiplication on complex number) on M is no greater than the normalized volume $Vol(P)$ of Newton polytope P. Moreover, the equality $E(M) = Vol(P)$ holds only when Einstein system restricted on every face has no solutions in algebraic torus (also it implies that all solutions are isolated). On the other hand, if there exists a face of polytope P such that Einstein system restricted on it has solution then this solution can be interpreted as a complex Ricci-flat invariant metric on some non-compact homogeneous manifold (called Inonu-Wigner contraction) associated with this face. In

the talk the series of flag manifolds $M_{n_1,n_2,n_3} = SO_{2(n_1+n_2+n_3)+1}/U_{n_1} \times U_{n_2} \times SO_{2n_3+1}$ with T-root system BC_2 will be considered. The faces of corresponding 5-dimensional Newton polytope $P(BC_2)$ were described by M. M. Graev. Using this description it will be shown that the number of complex invariant Einstein metrics on M_{n_1,n_2,n_3} is equal to exactly $Vol(P) = 132$ unless the parameters n_1, n_2, n_3 satisfy one of some algebraic equations which will be provided explicitly. Moreover, the family of (real) Ricci-flat Lorentzian manifolds will be constructed as Inonu-Wigner contractions of M_{n_1,n_2,n_3} by certain 2-dimensional faces of $P(BC_2)$.

Compatibility complexes for the Killing equation

Igor Khavkine
Prague, Czech Republic

Abstract: The Killing operator on a (pseudo-)Riemannian geometry (M, g) is $K_{ab}[v] = \nabla_a v_b + \nabla_b v_a$. The Killing equation $K[v] = 0$ is an overdetermined PDE and we will consider its compatibility complex K_i ($i \geq 0$), where $K_0 = K$ and any differential operator C satisfying $C \circ K_i = 0$ must factor as $C = C' \circ K_{i+1}$, for some differential operator C'. Relying on the "finite-type" property of K, I will discuss a practical construction of such a compatibility complex on geometries of sub-maximal symmetry, with examples coming from General Relativity. Prior to this work, there were very few examples with the full compatibility complex K_i or even just K_1 known.

Integrability and infinite hierarchies of symmetries or conservation laws

Nina Khor′kova
BMSTU, Moscow, Russia

Abstract: Integrable differential equations (no matter what definition of integrability is used) have infinite hierarchies of symmetries and/or conservation laws. We present wide classes of PDE also possessing infinite series of nonlocal symmetries or conservation laws. For example, any nontrivial conservation law of an l-normal PDE generates an infinite family of nonlocal conservation laws. The results are obtained within the framework of nonlocal theory of PDE.

Homotopy Poisson brackets and thick morphisms

Hovhannes Khudaverdian
Manchester, UK

Abstract: For an arbitrary manifold M, consider the supermanifolds ΠTM and ΠT^*M, where Π is the parity reversion functor. The supermanifold ΠTM has an odd vector field that can be identified with the de Rham differential d; functions on it can be identified with differential forms on M. The supermanifold ΠT^*M has a canonical odd Poisson bracket $[\,,\,]$ (the antibracket); functions on it can be identified with multivector fields on M. An arbitrary even function P on ΠT^*M which obeys the master equation $[P, P] = 0$ defines an even homotopy Poisson structure on the manifold M and an odd homotopy Poisson structure (the "higher Koszul brackets") on differential forms on M.

We construct a non-linear transformation from differential forms endowed with the higher Koszul brackets to multivector fields considered with the antibracket by using the new notion of a thick morphism of supermanifolds, a notion recently introduced.

Based on joint work with Th. Voronov.

Brackets and torsions

Yvette Kosmann-Schwarzbach
Paris, France

Abstract: The classical brackets of Schouten and of Nijenhuis are fundamental in geometry as well as in the theory of integrable systems. We shall introduce lesser known concomitants, the Haantjes torsion that serves to generalize the recursion operators of bihamiltonian systems and the Yano-Ako torsion that serves to define Frobenius manifolds, and we shall show how these two torsions are related, and related to the Nijenhuis torsion.

Integrability via Geometry

Boris Kruglikov
Tromsø, Norway

Abstract: The title summarizes a long project in collaboration with Eugene Ferapontov, Vladimir Novikov, Maciej Dunajski, Boris Doubrov, David Calderbank. I will mainly talk on the work joint with David Calderbank that was initiated almost four years ago and that has been finished only recently. We prove that Lax integrability for dispersionless determined nondegenerate PDE in 3D and 4D with quadric characteristic variety is equivalent to respectively Einstein-Weyl or self-duality property of the conformal structure associated to the variety on every solution of the PDE.

Anisotropic multi-phase filtration: geometrical approach

Alexei Kushner
Moscow, Russia

Abstract: A differential-geometric description of the models of two-phase filtration in a plane anisotropic porous media is proposed. The concept of the anisotropy metric is introduced. A generalization of the Buckley-Leverett model for anisotropic media is constructed.

Joint work with Atlas Akhmetzyanov and Valentin Lychagin.

On classification of the second order differential operators and differential equations

Valentin Lychagin
Tromsø, Norway

Abstract: We'll discuss a local classication of the second order linear differential operators and corresponding differential equations. Possibly Riemann ([1]) was the first who analyzed this problem and found curvature as an obstruction to transform differential operators of the second order to operators with constant coefficients. In dimension two Laplace ([2]) found "Laplace invariants" which are relative invariants of subgroup of rescaling transformations of unknown functions and Ovsyannikov ([3]) found the corresponding invariants. All invariants for hyperbolic equations in dimension two with respect to pseudogroup transformations included also diffeomorphisms of the base manifold were found by Ibragimov ([4]). For the case of ordinary differential operators it was done by Kamran and Olver ([5]) and for the case of linear ordinary differential equations of any order relative invariants were found by Wilczynski ([6]). We are going to consider the problem in all dimensions. The talk is based on joint work with Valeriy Yumaguzhin ([7]).

References

[1] B. Riemann, Gesammelte mathematische werke und avissenschaftlicher nachlass, XXII, pp. 357-370, Leipzig, Teubner, 1876.

[2] P.S. Laplace, Recherches sur le calcul integral aux differences partielles, in: Memoires de l'Acad emie royale des Sciences de Paris (1773/77), pp. 341-402; reprinted from P. S. Laplace, Oeuvres Compl etes, Vol. 9, Gauthier-Villars, Paris (1893).

[3] L.V. Ovsyannikov, Group properties of the Chaplygin equation, J. Appl. Mech. Tech. Phys., 3, 126-145 (1960).

[4] N. Kh. Ibragimov, Invariants of hyperbolic equations: solutions of the Laplace problem, Journal of Applied Mechanics and Technical Physics, Vol. 45, No. 2, pp. 158-166, 2004.

[5] N. Kamran and P. Olver, Equivalence of differential operators, SIAM J. Math. Anal. Vol. 20, No. 5, pp. 1172–1185, 1989.

[6] E.J. Wilczynski, Projective differential geometry of curves and ruled surfaces, Leipzig, Teubner, 1905.

[7] V. Lychagin and V. Yumaguzhin, Classification of the second order differential operators and differential equations, Geometry and Physics, v. 130, pp. 213-228, 2018.

Projective symmetries and superintegrable metrics

Giovanni Manno
Turin, Italy

Abstract: In this seminar I will discuss 2-dimensional metrics admitting vector fields whose local flow preserves geodesic curves (projective symmetries). We shall see how to construct quadratic first integrals of the geodesic flow of metrics admitting projective symmetries, also relating such integrals with the notion of projective equivalence. Then we shall see that metrics with three projective symmetries are superintegrable, i.e., the space of the quadratic integrals of their geodesic flow is 4-dimensional. Time permitting, we shall give a classification of metrics admitting at least a projective symmetry.

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q-difference Painleve equations: symmetries and solutions

Andrei Marshakov
Moscow, Russia

Abstract: q-difference Painleve equations are considered as deautonomization of cluster integrable systems, where discrete flows are generated by quiver mutations. Their symmetries are given by extended affine Weyl groups, and solutions can be written in terms of Nekrasov functions for $5d$ supersymmetric gauge theories.

Symmetries of asymptotically flat spaces and gravitational memory effect

Luigi Martina
Lecce, Italy

Abstract: We review the conformal Carroll group and consider its relation with the Bond-Metzler-Sachs group for asymptotically flat spacetimes. Since the Carroll group emerges as the isometry group of a class of plane gravitational waves, we discuss some specific examples, which we use in order to illustrate the so-called gravitational memory effect.

On symmetries of the Gibbons-Tsarev equation

Michal Marvan
Opava, Czech Republic

Abstract: We present explicit formulas for hierarchies of conservation laws and nonlocal symmetries of the Gibbons-Tsarev equation.

This is a joint work with I.S. Krasil′shchik, P. Blaschke et al.

Integrable magnetic geodesic flows on 2-torus

Andrey Mironov
Novosibirsk, Russia

Abstract: The only one example has been known of magnetic geodesic flow on the 2-torus which has a polynomial in momenta integral independent of the Hamiltonian. In this example the integral is linear in momenta and corresponds to a one parametric group preserving the Lagrangian function of the magnetic flow. In the talk the problem of integrability on one energy level will be considered. This problem can be reduced to a remarkable Semi-hamiltonian system of quasi-linear PDEs and to the question of existence of smooth periodic solutions for this system. Our main result states that the pair of Liouville metric with zero magnetic field on the 2-torus can be analytically deformed to a Riemannian metric with small magnetic field so that the magnetic geodesic flow on an energy level is integrable by means of a quadratic in momenta integral. Thus our construction gives a new example of smooth periodic solution to the Semi-hamiltonian (Rich) quasi-linear system of PDEs.

The result were obtained with Michael Bialy and Sergey Agapov.

Tropical limit of soliton solutions, Yang-Baxter maps and beyond

Folkert Müller-Hoissen
Göttingen, Germany

Abstract: The "tropical limit" of a soliton solution of an integrable equation in two space-time dimensions consists of a graph, representing the motion of wave crests, together with values of the dependent variable along its segments. This associates with the wave solution a classical point particle picture, in which free particles (only) interact at certain events (points in space-time). For example, in case of matrix KdV solitons, at such an event the matrix data are related by a Yang-Baxter map, a nonlinear solution of the ("quantum") Yang-Baxter equation. This solution has been obtained previously (up to a missing factor) from the 2-soliton solution of the matrix KdV equation [1]. A known relation between Yang-Baxter maps and "multidimensional consistency" relates them to discrete integrable equations [2].

In two-dimensional integrable quantum field theory models, the Yang-Baxter equation expresses factorization of the multi-particle scattering matrix. This means that the latter decomposes into 2-particle interactions. Similarly, we think of a multi-soliton solution (of some integrable equation) also as being composed of 2-soliton interactions. But because of the wave nature of solitons, there are no definite events at which the interaction takes place. However, the tropical limit takes the waves to "point particles" and then indeed determines events at which an interaction occurs, and where we find a Yang-Baxter map (and perhaps related structures) at work. More precisely, the Yang-Baxter equation expresses that, in a 3-particle interaction, two different sequences of the three 2-particle interactions lead to the same result. This finds a natural explanation in the tropical limit of a 3-soliton solution.

Besides matrix KdV, we explored the matrix KP equation [3] and its matrix Boussinesq reduction [5] in detail. Here a Yang-Baxter map, acting along a tropical limit graph, appears to be insufficient to describe all possible soliton interactions in the tropical limit, and additional structures enter the stage [4].

This talk presents an overview of some of our recent results [3-5].

References

[1] A. Veselov, "Yang-Baxter maps and integrable dynamics", *Phys. Lett. A* **314** (2003) 214.

[2] V. Papageorgiou, A. Tongas and A. Veselov, "Yang-Baxter maps and symmetries of integrable equations on quad-graphs", *J. Math. Phys.* **47** (2006) 083502.

[3] A. Dimakis and F. Müller-Hoissen, "Matrix KP: tropical limit and Yang-Baxter maps", preprint arXiv:1708.05694 [nlin.SI].

[4] A. Dimakis and F. Müller-Hoissen, "Matrix Kadomtsev-Petviashvili equation: tropical limit, Yang-Baxter and pentagon maps", Theor. Math. Phys. **196** (2018) 1164.

[5] A. Dimakis, F. Müller-Hoissen and X.-M. Chen, "Matrix Boussinesq solitons and their tropical limit", arXiv:1805.09711 [nlin.SI].

Lie remarkable systems of PDEs characterized by affine and projective transformations

Francesco Oliveri
Messina, Italy

Abstract: Within the framework of inverse Lie problems, we give some non-trivial examples of coupled Lie remarkable equations, i.e., classes of partial differential equations that are uniquely characterized by their Lie point symmetries. In particular, we find second and third order systems of partial differential equations in correspondence to affine and projective transformations.

Generalized symmetries and conservation laws of $(1+1)$-dimensional Klein-Gordon equation

Stanislav Opanasenko
St. John's, Canada

Abstract: We give an explicit form of the algebra of generalized symmetries of the $(1+1)$-dimensional Klein–Gordon equation in terms of a quotient subalgebra of the universal enveloping algebra of the essential Lie invariance algebra thereof and use it for computing the space of local conservation laws of this equation.

Integrability of the Gibbons-Tsarev system

Maxim Pavlov
Moscow, Russia

Abstract: We present wide classes of ansatzes for construction of solutions of the GT system.
Corresponding solutions are parameterised by an arbitrary number of constants.

Thermodynamics of submerged jets: exact solutions of the Navier-Stokes equations

Michael Roop
Moscow, Russia

Abstract: We use symmetry methods to construct exact solution of the Navier-Stokes equations with heat balance equation, which corresponds to a submerged jet - jet of fluid or gas issued from a point source into the non-restricted medium of the same kind - and describes the distribution of temperature in the jet besides. This solution is invariant to subalgebra of the symmetry algebra of the system. We use normalizers to find a proper subalgebra for reduction.

Nonlinear waves in layered media: solutions of the KdV-Burgers equation

Alexey Samokhin
Moscow, Russia

Abstract: The KdV-Burgers equation is used to model a behavior of a KdV-soliton which, while moving in non-dissipative medium encounters a barrier with dissipation. The layered media consist of layers with both dispersion an dissipation and layers without dissipation. In the latter case the waves are described by the KdV equation, while in the former - by the Kdv-Burgers one.

The dissipation results in reducing the soliton amplitude/velocity, and a reflection and refraction occur at the boundary(s) of a dissipative layer. In the case of a finite width barrier on the soliton path,

after the wave leaves the dissipative barrier it retains a soliton form and a reflection wave arises as small and quasi-oscillations (a breather). Hence a soliton solution of the KdV equation, meeting a layer with dissipation, transforms somewhat similarly to a ray of light in the air crossing a semi-transparent plate.

Another case models a passage from non-dissipative half-space to a dissipative one. It is natural to expect each solution to behave as the one of the KdV at the first half-space and as solution of KdV-B at the second one. The process of transition from the soliton to the correspondent solution of the KdV-B is predictable. The transient wave in a dissipative media becomes a solitary shock which loses speed and decays to become nonexistent at $t \to +\infty$; and a reflected wave is seen in the non-dissipative half-space.

Other traveling wave solutions are also studied within this model.

Reduction in soliton hierarchies and special points of classical r-matrices

Taras Skrypnyk
Milan, Italy

Abstract: We propose the most general approach to construction of the U-V pairs of hierarchies of soliton equations in two dimensions based on the theory of classical non-skew-symmetric r-matrices with spectral parameters and infinite-dimensional Lie algebras. We show that "reduction" in integrable hierarchies is connected with "special points" of classical r-matrices in which they become singular or degenerated. We prove that "Mikhailov's reduction" or reduction with the help of automorphism is a partial case of our construction.

Traces of Tractor Calculus in Sub-Riemannian Geometry

Jan Slovák
Brno, Czech Republic

Abstract: I will present an approach to sub-Riemannian geodesics motivated by tractor calculus.

In particular, there is a nice system of equations coupling the fields in the sub-Riemannian distribution with fields in its annihilator, and its solutions describe the normal geodesics locally.

I will also try to illustrate how these equations work on some examples.

Joint work in progress with Rod Gover.

Beyond recursion operators: Haantjes algebras

Giorgio Tondo
Trieste, Italy

Abstract: I will illustrate in detail the notion of Haantjes algebra, recently introduced to generalize the very successful Magri's theory of recursion operators for soliton equations. Haantjes algebras are associative and commutative algebras of operators with vanishing Haantjes torsion, over differentiable manifolds. In such a context, the powers of a recursion operator are replaced by a distinguished basis of a Haantjes algebra. The case of semisimple Haantjes algebras over symplectic manifolds, leading to separation of variables for Hamilton-Jacobi equations associated with separable Hamiltonian systems, will be analyzed. Also, some examples of non semisimple Haantjes algebras coming from the theory of hydrodynamic-type systems, will be presented.

Symmetries and conservation laws for a generalization of Kawahara equation

Jakub Vašíček
Opava, Czech Republic

Abstract: In this talk we study formal and generalized symmetries and local conservation laws for a fifth order nonlinear evolutionary partial differential equation in two independent and one dependent variable which generalizes the Kawahara equation. In particular, we establish nonexistence of formal symmetries of rank greater than 13 and give a complete description of generalized symmetries and local conservation laws for the equation under study.

Nonlocal Jacobi identity: a geometric approach

Alexander Verbovetsky
Moscow, Russia

Abstract: Nonlocal Hamiltonian operators can be understood as Bäcklund transformations between tangent and cotangent coverings. In this talk, we will discuss the nonlocal Jacobi identity, including relation to Dirac structures.

Joint work with Joseph Krasil′shchik.

Projective-geometric aspects of WDVV equations

Raffaele Vitolo
Lecce, Italy

Abstract: In this talk we will uncover new projective-geometric aspects of the WDVV equations in the light of recent developments about the projective-geometric nature of their (bi)-Hamiltonian formalism.

Joint work with E. Ferapontov and M. Pavlov

Coverings and nonlocal symmetries of Lax-integrable PDEs

Petr Vojčák
Opava, Czech Republic

Abstract: We consider four three-dimensional Lax-integrable equations: (1) the rdDym equation $u_{ty} = u_x u_{xy} - u_y u_{xx}$, (2) the Pavlov equation $u_{yy} = u_{tx} + u_y u_{xx} - u_x u_{xy}$, (3) the universal hierarchy equation $u_{yy} = u_t u_{xy} - u_y u_{tx}$, and (4) the modified Veronese web equation $u_{ty} = u_t u_{xy} - u_y u_{tx}$.

For each equation, expanding the known Lax pairs in formal series in the spectral parameter, we construct two differential coverings and completely describe the nonlocal symmetry algebras associated with these coverings. For all four pairs of coverings, the obtained Lie algebras of symmetries manifest similar (but not identical) structures; they are (semi)direct sums of the Witt algebra, the algebra of vector fields on the line, and loop algebras, all of which contain a component of finite grading. We also discuss actions of recursion operators on shadows of nonlocal symmetries.

This is the joint work with Hynek Baran, Iosif S. Krasil′shchik and Oleg I. Morozov.

Classification of 3rd - order linear differential equations

Valery Yumaguzhin
Pereslavl'-Zalesskiy

Abstract: In this talk, we consider generic 3rd - order scalar linear differential equations on a two dimensional manifold. We give a local classification of these equations up to diffeomorphisms of the manifold.

Let M be a 2 - dimensional manifold and $\xi : M \times \mathbb{R} \to M$ be a trivial line bundle. We identify its module of smooth sections $C^\infty(\xi)$ with a module of smooth functions $C^\infty(M)$. By $\mathrm{Diff}_k(M)$ we denote the left module of scalar linear differential operators of order $\leq k$ acting in $C^\infty(\xi)$.

Considering equations have the form $A(f) = 0$, where $A \in \mathrm{Diff}_3(M)$ is a generic operator. In local coordinates x^1, x^2 of M, it has the form

$$A = a^{ijk}\partial_{x^i}\partial_{x^j}\partial_{x^k} + a^{ij}\partial_{x^i}\partial_{x^j} + a^i\partial_{x^i} + a^0.$$

Let $\tau : T(M) \to M$ and $\tau^* : T^*(M) \to M$ be tangent and cotangent bundles. Then $\sigma_A = a^{ijk}\partial_i \odot \partial_j \odot \partial_k \in C^\infty(S^3\tau)$ is the symbol of A.

It follows from the generality condition of operator A that there are a unique symmetric linear connection ∇_{T^*} in the bundle τ^* (*Chern connection*) and a unique differential 1-form $\theta \in C^\infty(\tau^*)$ such that

$$\nabla_{T^*}(\sigma_A) = \theta \otimes \sigma_A.$$

Proposition 1. *Let f be everywhere nonzero smooth function in M. Then Chern connections for operators A and $f \cdot A$ are the same.*

Let ∇_ξ be a trivial connection in the bundle ξ. It is well known that the connections ∇_{T^*} and ∇_ξ generate a natural decomposition $J^k\xi = \oplus_{m=0}^k S^m T^*(M)$ such that $j_k(f) = \{D_m(f)\}_{0 \le m \le k}$, $\forall f \in C^\infty(M)$, where operators D_m are defined by ∇_{T^*} and ∇_ξ.

Theorem 2. *Let $A \in \mathrm{Diff}_3(M)$ be a generic operator, ∇_{T^*} be its Chern connection and ∇_ξ be the trivial connection in the bundle ξ. Then there is a natural decomposition of the operator A*

$$A = \Delta_3 + \Delta_2 + \Delta_1 + \Delta_0,$$

such that

1. $\Delta_k \in \mathrm{Diff}_k(M)$, $k = 3, 2, 1, 0$.
2. $\Delta_3(1) = \Delta_2(1) = \Delta_1(1) = 0$, *and* $\Delta_0 = a^0$.
3. *The symbol σ_{Δ_2} of the operator Δ_2 is contravariant pseudo-Riemannian metric. Then $g_{\Delta_2}(\Delta_1, \Delta_1) \neq 0$, where g_{Δ_2} is a covariant metric corresponding to σ_{Δ_2}.*
4. *Scalar invariants $I^1 = a^0$ and $I^2 = g_{\Delta_2}(\Delta_1, \Delta_1)$ are functionally independent.*

Let $\mathcal{G}(M)$ be a pseudogroup of local diffeomorphisms of M.

Proposition 3. *Let $\tilde{A} \in \mathrm{Diff}_3(M)$ be another generic operator. Then the operators A and $\tilde{A}$ are locally equivalent with respect to $\mathcal{G}(M)$ if and only if their expressions in coordinates I^1, I^2 are the same.*

Theorem 4. *Differential equations, given by generic differential operators $A \in \mathrm{Diff}_3$ are locally equivalent with respect to $\mathcal{G}(M)$ if and only if their normalized operators $\dfrac{1}{I^2}A$ are locally equivalent with respect to $\mathcal{G}(M)$.*