Automatic Determination of Optimal Systems of Lie Subalgebras

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Abstract

It is known that a Lie group of symmetries (generating a Lie algebra) admitted by a partial differential equation can be used for determining group invariant solutions. By taking different subgroups of the principal Lie group, different sets of group invariant solutions can be characterized, most of which are connected by group transformations. Since a Lie group G (or Lie algebra $\mathfrak{g}$) usually contains infinitely many subgroups (or subalgebras) of the same dimension, a classification of them up to some equivalence relation is necessary. Following Ovsiannikov [1], two subalgebras $\mathfrak{g}_1$ and $\mathfrak{g}_2$ of a given finite-dimensional Lie algebra $\mathfrak{g}$ are equivalent if one can find some element g in the Lie group generated by $\mathfrak{g}$ such that $\mathrm{Ad}_g(\mathfrak{g}_1) = \mathfrak{g}_2$, where Ad_g is the adjoint representation of g on $\mathfrak{g}$.

A family of s-dimensional subalgebras $\{\mathfrak{g}_\alpha\}$ is an optimal system if it contains only inequivalent s-dimensional Lie subalgebas, and any s-dimensional subalgebra is equivalent to some element of the family.

An effective computer algebra program allowing for the automatic derivation of optimal systems of Lie subalgebras is presented, and some results in the literature amended.

References

[1] L.V. Ovsiannikov. Group Analysis of Differential Equations. Academic Press, New York, 1982.