# Tropical limit of soliton solutions, Yang-Baxter maps and beyond

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**Abstract:**
The "tropical limit" of a soliton solution of an integrable equation in two space-time dimensions consists of a graph, representing the motion of wave crests, together with values of the dependent variable along its segments. This associates with the wave solution a classical point particle picture, in which free particles (only) interact at certain events (points in space-time). For example, in case of matrix KdV solitons, at such an event the matrix data are related by a Yang-Baxter map, a nonlinear solution of the ("quantum") Yang-Baxter equation. This solution has been obtained previously (up to a missing factor) from the 2-soliton solution of the matrix KdV equation [1]. A known relation between Yang-Baxter maps and "multidimensional consistency" relates them to discrete integrable equations [2].

In two-dimensional integrable quantum field theory models, the Yang-Baxter equation expresses factorization of the multi-particle scattering matrix. This means that the latter decomposes into 2-particle interactions. Similarly, we think of a multi-soliton solution (of some integrable equation) also as being composed of 2-soliton interactions. But because of the wave nature of solitons, there are no definite events at which the interaction takes place. However, the tropical limit takes the waves to "point particles" and then indeed determines events at which an interaction occurs, and where we find a Yang-Baxter map (and perhaps related structures) at work. More precisely, the Yang-Baxter equation expresses that, in a 3-particle interaction, two different sequences of the three 2-particle interactions lead to the same result. This finds a natural explanation in the tropical limit of a 3-soliton solution.

Besides matrix KdV, we explored the matrix KP equation [3] and its matrix Boussinesq reduction [5] in detail. Here a Yang-Baxter map, acting along a tropical limit graph, appears to be insufficient to describe all possible soliton interactions in the tropical limit, and additional structures enter the stage [4].

This talk presents an overview of some of our recent results [3-5].

## References

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