Presymplectic gauge PDEs and Batalin-Vilkovisky quantization

Maxim Grigoriev

Lebedev Physical Institute
&
Institute for Theoretical and Mathematical Physics, Lomonosov MSU

Based on: 2109.05596, 2008.11690, 1903.02820, 1606.07532, 1312.5296, 1204.1793, 1012.1903, 0905.0547, 0605089, 0602166
Collaborations with Glenn Barnich, Konstantin Alkalaev, Alexei Kotov, and Slava Gritsaenko
Also: Alik Verbovetsky, Alexey Sharapov

Alexandre Vinogradov Memorial Conference
Diffieties, Cohomological Physics, and Other Animals

Introduction

- Despite being rather abstract Vinogradov's geometry of PDEs appears quite natural form the field theory perspective! Covariant phase space methods, homlogical methods, Noether method, etc.)

- Becomes especially attractive in the case of BV-BRST approach to gauge theories: homological resolutions of the equation manifold, physical quantities as local BRST cohomology classes), distinction between equations (Lagrangian) and their solutions. For instance:

The International Conference on

Secondary Calculus and Cohomological Physics

Moscow
August 24 - August 31, 1997

Organized by

Diffiety Institute of the Russian Academy of Natural Sciences
in collaboration with
M.V. Lomonosov Moscow State University
Istituto Italiano per gli Studi Filosofici
with support from the
ISTITUTO ITALIANO PER GLI STUDI FILOSOFICI
RUSSIAN FOUNDATION FOR BASE RESEARCH
INTERNATIONAL CENTER FOR FUNDAMENTAL PHYSICS (Moscow)

Scientific Committee

I. Batalin (Moscow), L. Bonora (Triest), R. Bryant (Durham, NC), M. Dubois-Violette (Orsay), M. Fliess (Paris), P. Griffiths (Princeton, NJ), M. Henneaux (Brussels), V. Kozlov (Moscow), J. Krasil'shchik (Moscow), G. Marmo (Naples), B. Sadovnikov (Moscow), J. Stasheff (Chapel Hill, NC), M. Talon (Paris), I. Tyutin (Moscow), C. Viallet (Paris), A. Vinogradov (Salerno-Moscow), B. Voronov (Moscow)

Local Committee:

V. Kozlov, B. Sadovnikov, J. Krasil'shchik, A. Samokhin, M. Vinogradov (Moscow)

- From physics point of view: theories of fundamental interactions (Gravity, YM, Strings, M-Theory, Higher-spin theories ...) are inevitably gauge theories. We are mostly intrested in Lagrangian gauge theoires!

- BV formalism on jet-bundles: *Henneaux, Barnich, Brandt,........* Applies to variational PDEs on jet-bundles. Moreover, analyzing local BRST cohomology has lead byond jet-bundle objects such as transgression formulas, generalized connections, etc. *Stora, Brand, Baulieu,.....*

- Both from the fundamental perspective and applications in gravity, asymptotic symmetries, holography, higher spin gauge theories, string field theory, etc. it is highly desierable to develop a geometrically invariant approach to local gauge theories, analagous to the *Vinogradov* approach to PDEs

- Full-scale BV extension of the concept of diffiety remaines somewhat obscured
  - BV is essentially Lagrangian (defined on jet-bundles). One needs BV extension of not necessarily variational PDEs
  - We are often intersted in variational PDEs. How (BV) Lagrangian is encoded in term of (BV extension of) diffiety?

- It turns out that a bridge between BV formalism and the invariant geometrical approach to PDEs becomes manifest using the *Alexandrov-Kontsevich-Schwartz-Zaboronsky 1994* (AKSZ)-like framework. This was originally proposed as an elegant BV formulation of topologiacal models. Somewhat similar approach (in terms of free differential algebras (FDA)) was independently developped by *M.Vasiliev* in the context of higher-spin theories. It is also worth mentioning FDA approach to supergravity by *D'Auria, Fre,...*.

PDEs and jet-bundles

Fiber-bundle $\mathcal{F} \to X$ (global aspects are not discussed):
base space (independent variables or space-time coordinates): x^a, $a = 1, \ldots, n$.
Fiber: (dependent variables or fields ϕ^i)

Jet-bundle:
A point of J^n is a pair $(x, [\sigma])$, where $[\sigma]$ is an equivalence class of sections $\sigma : X \to \mathcal{F}$ such that their partial derivatives at x coincide to order n.

One can use x^i, and values of derivatives as coordinates:

$J^0(\mathcal{F})$: $x^a, \phi^i,$ $J^1(\mathcal{F})$: $x^a, \phi^i, \phi^i_a\,,$ $J^2(\mathcal{F})$ $x^a, \phi^i, \phi^i_a, \phi^i_{ab}\,,$ $\ldots$

Projections:

$$\ldots \to J^N(\mathcal{F}) \to J^{N-1}(\mathcal{F}) \to \ldots \to J^1(\mathcal{F}) \to J^0(\mathcal{F}) = \mathcal{F}$$

Useful to work with $\mathcal{J} := \mathcal{J}^\infty$ (projective limit).

A local function is a pull-back of a function from $J^N(\mathcal{F})$ for some N. i.e. it depends on only a finite number of the coordinates. A local function $f = f(x, \phi, \phi_a, \phi_{ab} \ldots)$ can be evaluated on a section $\sigma : X \to \mathcal{F}$ as

$$f(\sigma) := f(x, \sigma^*(\phi^i), \partial_a \sigma^*(\phi^i), \ldots)$$

Total derivative: (imitates the action of standard partial derivative)

$$D_a := \frac{\partial}{\partial x^a} + \phi^i_a \frac{\partial}{\partial \phi^i} + \phi^i_{ab} \frac{\partial}{\partial \phi^i_a} + \dots$$

Main property:

$$\partial_a(f(\sigma)) = (D_a f)(\sigma)\,.$$

Total derivatives generate Cartan distribution.

Similarly one defines local forms. These are forms that can be obtained by pullback from finite jets.

Space-time differentials dx^a. Horizontal differential:

$$d_{\mathsf{h}} \equiv dx^a D_a\,, \qquad d_{\mathsf{h}}^2 = 0\,.$$

A system of partial differential equations (PDE) is a collection of local functions on $\mathcal{J}$

$$E_\mu[\phi, x]\,.$$

The equation manifold (stationary surface): $E \subset \mathcal{J}$ singled out by: (prolonged equation)

$$D_{a_1} \dots D_{a_l} E_\mu = 0\,, \qquad l = 0, 1, 2, \dots$$

understood as the algebraic equations in $\mathcal{J}$.

D_a are tangent to E and hence restricts to E. So do the differentials d_h and d_v. $D_a|_E$ determine a dim-n involutive distribution – Cartan distribution.

Definition: *[Vinogradov] PDE is a manifold E equipped with an involutive Cartan distribution $\mathcal{C} \subset TE$.*

In addition one typically assumes regularity, constant dimension, and that E is a bundle over the spacetime and can be locally embedded into a jet-bundle.

PDEs are isomorphic when the respective distributions are.

Differential forms on E form the variational bicomplex of E. Note that in general $H^{k,\bullet}(d_\mathsf{h}) \neq 0$, e.g. $H^{n-k,0}$, degree-k conservation laws.

For $n = 0$ PDEs are just usual manifolds.

Towards gauge PDEs

Nonlagrangian version of BV: forget about symplectic structure and keep Cartan‘, BRST differential, ghost degree. *Barnich, M.G., Semikhatov, Tipunin 2004*, *Lyakhovich, Sharapov, 2004...*

Althogh gaueg PDE it's a simple notion and examples were in the literature (mostly in the context of topological models or higher spin theories) the general concept appeared under the name of "parent formalism" *Barnich, M.G. 2010*
Idea: reformulate BV as an AKSZ sigma model. In the case of PDE the minimal equivalent formulation of this type has diffiety as a target space. This way one arrives at BV-BRST extensions of diffieties.

More refined and explicit definition of gauge PDE was in *M.G., Kotov, 2019*. Not a "direct product" of PDE and Q-manifold concpets

Q-manifolds

Def. Q-manifold (M, Q) is a $\mathbb{Z}$-graded supermanifold M equipped with the odd nilpotent vector field of degree 1, i.e.

$$Q^2 = 0\,, \qquad |Q| = 1\,, \quad \mathrm{gh}(Q) = 1$$

Example: Odd tangent bundle: $(T[1]X, d_X)$. If θ^a are coordinates on the fibres of $T[1]M$ in the basis $\frac{\partial}{\partial x^a}$:

$$d_X := \theta^a \frac{\partial}{\partial x^a}$$

Example: CE complex $(\mathfrak{g}[1], Q)$. If $\mathfrak{g}$ is a Lie algebra then $\mathfrak{g}[1]$ is equipped with Q structure. If c^α are coordinates on $\mathfrak{g}[1]$ in the basis e_α then

$$Q = c^\alpha c^\beta U^\gamma_{\alpha\beta} \frac{\partial}{\partial c^\gamma}\,, \qquad [e_\alpha, e_\beta] = U^\gamma_{\alpha\beta} e_\gamma$$

$\phi : (M_1, Q_1) \to (M_2, Q_2)$ is a Q-map if $\phi^* \circ Q_2 = Q_1 \circ \phi^*$

Example: $(V[1](M), Q)$ where $V(M)$ Lie algebroid. Indeed generic Q of degree 1 locally reads as:

$$Q = c^\alpha R_\alpha - \frac{1}{2} c^\alpha c^\beta U^\gamma_{\alpha\beta}(z) \frac{\partial}{\partial c^\gamma}$$

R_α gives anchor, $U^\gamma_{\alpha\beta}$ bracket, $Q^2 = 0$ encodes compatibility.

Gauge PDE in $n = 0$ (trivial Cartan distribution) is a Q-manifold $(\mathcal{E}, Q)$ that is equivalent to a nonnegatively graded one.

If only ghost degree $0, 1$ variables are present then it is just a Lie algebroid.

Proposition: [AKSZ, 1994] Let $p \in \mathcal{E}$ and $Q|_p = 0$ then $T_p\mathcal{E}$ is an L_∞ algebra.

Important feature: although this is an intrinsic definition ($\mathcal{E}$ is not embedded into some "jet space") there are infinitely many Q-manifolds representing the same gauge PDE.

Equivalence of Q-manifolds:

Idea: restrict to local analysis and suppose that (M, Q_M) can be represented as a product Q-manifold:

$$M = N \times T[1]\mathbb{R}^k\,, \qquad Q_M = Q_N + d_{T[1]\mathbb{R}^k}$$

then (M, Q_M) and (N, Q_N) are equivalent. Q-manifold of the form $(T[1]V, d_{T[1]V}$ is caled contractible.

In coordinates:

$$Q_M = Q_N + v^\alpha \frac{\partial}{\partial w^\alpha}\,, \qquad Q_N = q^i(\phi)\frac{\partial}{\partial \phi^i}\,.$$

Often one can find "minimal" Q-manifold describing a given equation. This gives a minimal model of the respective L_∞ algebra.

In general (taking global geometry into account) homotopy equivalence of Q-manifolds.

In the context of gauge theories: w^α, v^α – are known as "generalized auxiliary fields" *Henneaux, 1990* (in the Lagrangian case).

Def. *[Kotov, Strobl]* Localy trivial bundle $\pi : E \to M$ of Q-manifolds is called Q-bundle if π is a Q-map. Section $\sigma : M \to E$ is called Q-section if it's a Q-map.

In general, $\pi : E \to M$ is not a locally trivial Q-budle.

Indeed, although locally $E \cong M \times F$ (product of manifolds) in general Q is not a product Q-structure of Q_F and Q_M.

Example: let $\pi_X : E \to X$ be a fibered bundle then
$\pi = d\pi_X : (T[1]E, d_E) \to (T[1]X, d_X)$ is a Q-bundle.

Def. (M, Q) is called an equivalent reduction of (M', Q') if (M', Q') is a locally trivial Q-bundle over (M, Q) with a contractible fiber and (M', Q') admits a global Q-section.

This generates an equivalence relation for Q-manifolds.

PDE as a Q-bundle

Consider PDE $(E_X, \mathcal{C})$, E_X is a bundle $\pi_X : E_X \to X$ over space-time X, $\mathcal{C} \subset TE_X$ is a Cartan distribution, π_X induces an isomorphism $C_p E_X \to T_{\pi_X(p)} X$, $p \in E_X$.

In particular, total derivatives D_a satifiy $d\pi_X(D_a) = \frac{\partial}{\partial x^a}$, where x^a are local coordinates on X.

Algebra of horizontal differential forms can be seen as functions on $C[1]$. $\mathcal{C}$ gives rise to horizontal differential d_{h}. In coordinates:

$$d_{\mathsf{h}} = \theta^a D_a \qquad (\theta^a \equiv dx^a)$$

Pulling back $(E_X, \mathcal{C})$ to a bundle $E_{T[1]X}$ over $T[1]X$ gives a Q-bundle $\pi : (E_{T[1]X}, d_{\mathsf{h}}) \to (T[1]X, d_X))$.

This Q-bundle $\pi : (E_{T[1]X}, d_{\mathsf{h}}) \to (T[1]X, d_X))$ encodes all the information about the starting point PDE $(E_X, \mathcal{C})$.

For instance, in terms of $(E_X, \mathcal{C})$ solution is by definition a section $\sigma : X \to E_X$ which is tangent to $\mathcal{C}$. Seen as a section of $E_{T[1]X}$ σ is a Q-section and other way around. If ψ^A are local coordinates on the fibres the section is parametrized by $\sigma^A(x) = \sigma^*(\psi^A)$ Q-map condition $d_X \circ \sigma^* = \sigma^* \circ d_{\mathsf{h}}$ gives:

$$\frac{\partial}{\partial x^a}\sigma^A(x) = \Gamma^A_a(\sigma(x), x)\,, \qquad d_{\mathsf{h}} = \theta^a D_a = \theta^a(\frac{\partial}{\partial x^a} + \Gamma^A_a(\psi, x)\frac{\partial}{\partial \psi^A})$$

also known as "unfolded" representation *Vasiliev*. In particular, fields of the unfolded form are coordinates on the equation manifold (stationary surface).

Note that Q-bundles originating from PDEs are quite special: $\mathbb{Z}$-grading (ghost degree) originates from just the space-time form degree (the only nonzero degree coordinates are θ^a).

Gauge PDEs

In terms of Q-bundles PDEs can be defined as Q-bundles over $T[1]X$ with horizontal $\mathbb{Z}$-grading. The extension to the case of gauge systems is surprisingly straitforward: just forget about horizontality

Def. Gauge pre-PDE is a Q-bundle $(E_{T[1]X}, Q)$ over $(T[1]X, \mathrm{d}_X)$

Equivalence of Q-manifolds exends to Q-bundles over $T[1]X$, giving the notion of equivalent reduction and equivalence of gauge pre-PDEs. Notion of gauge pre-PDE is too wide:

gauge PDE: equivalent to nonnegatively graded, realizable in term of superjet bundle in a regular way. In applications we often (but not always!) also want gauge PDE to be proper – i.e. that all the gauge symmetries of the underlying PDE are taken into account by Q.

Equations of motion and gauge symmetries

Solutions: $\sigma : T[1]X \to E_{T[1]X}$ is a solution if

$$d_X \circ \sigma^* = \sigma^* \circ Q$$

Gauge transformations:

$$\delta\sigma^* = d_X \circ \epsilon^*_\sigma + \epsilon^*_\sigma \circ Q,$$

Gauge parameter: $\epsilon^*_\sigma : \mathcal{C}^\infty(E_{T[1]X}) \to \mathcal{C}^\infty(T[1]X)$,

$$\mathrm{gh}(\epsilon^*_\sigma) = -1, \quad \epsilon^*_\sigma(fg) = \epsilon^*_\sigma(f)\sigma^*(g) \pm \sigma^*(f)\epsilon^*_\sigma(g)$$

Gauge for gauge symmetries . . .

Batalin-Vilkovisky formulation (at the level of equations of motion)

Fields Ψ^A (include genuine fields ϕ^i, ghosts c^α, antighosts π_μ, antifields $\mathcal{P}_a$, ...). Jet-bundle with coordinates $\Psi^A_{b_1\ldots}, x^a, \theta^a$

Horizontal differential: $d_{\mathsf{h}} = \theta^a D_a$

BV-BRST differential: $s, \mathrm{gh}(s) = 1, s^2 = 0, [d_{\mathsf{h}}, s] = 0$

This can be taken as a definition of gauge theory *Barnich, M.G., Semikhatov, Tipunin 2004*.

In particualr EOMs, gauge symmetries, etc can be expressed in terms of s and degree.

Independent approach *Lyakhovich, Sharapov, 2004*

Consider BV jet-bundle as a Q-bundle over $T[1]X$ with $Q = d_{\mathsf{h}} + s$. If we restrict to bidegree preserving maps – Q-bundle description of the original gauge theory (almost trivial).

Less trivial – just total degree (form degree + BV ghost numebr) preserving maps. Follows from *Barnich, M.G. 2010*.

Formalism encodes BV as a particular case and hence all reasonable gauge theories. Justifies definition.

Example: Maxwell equation as a gauge PDE

Trivial bundle $T[1]X \times M$, Fiber coordinates:

$$C, \quad \mathrm{gh}(C) = 1\,, \qquad F_{a|b}, \quad F_{a|b_1 b_2}, \quad \ldots \quad F_{a|b_1\ldots b_l} \quad \ldots \qquad \mathrm{gh}(F_{\ldots}) = 0$$

Irreducible tensors, symmetric in second group, traceless:

$$Qx^a = \theta^a, \quad Q\theta^a = 0, \quad QC = \frac{1}{2}F_{ab}\theta^a\theta^b, \quad QF_{a|b} = \theta^c F_{a|bc}, \quad \ldots$$

Equations of motion (promoting C, F to fields $\sigma^*(C) + A_a(x)\theta^A$, $\sigma^*(F_{\ldots}) = F_{\ldots}(x)$ *M.Vasiliev*

$$\partial_a A_b - \partial_b A_a = F_{a|b}\,, \qquad \partial_c F_{a|b} = F_{a|bc}\,, \quad \ldots$$

taking a trace of the 2nd gives $\eta^{bc}\partial_a F_{b|c} = 0$.

Parent formulation

Given a gauge PDE $(E_{T[1]X}, Q)$ consider a new one $(E^P_{T[1]X}, Q^P)$, where

$$E^P_{T[1]X} = SJ^\infty(E_{T[1]X}, Q))$$

$$Q^P \quad \text{– prolongation of } Q$$

Proposition: Parent formulation is equivalent to the original. Local proof *Barnich, M.G. 2010*. Fully global – work in progress.

Reparametrization invariance and AKSZ sigma models

Suppose that $(E_{T[1]X}, Q)$ is a locally trivial Q-bundles. Restrict to local analysis. Then

$$(E_{T[1]X}, Q) = (T[1]X, d_X) \times (F, Q_F)$$

Gauge PDEs of this type are known as AKSZ sigma models.

In higher dimension: local triviality = reparmetrization invariance (in the context of BRST cohomology this was known as a posibility to eliminate d_{h} through change of variables, *Brandt, Dragon; Barnich, Brandt, Henneaux (1993))*)

In particular, any reparametrization-invariant guage theory (e.g. gravity) can be locally represented as AKSZ sigma model *Barnich, M.G. 2010*

Example: zero-curvature equation

Take $E_{T[1]X} = (T[1]X, d_X) \times (\mathfrak{g}[1], Q)$, where $\mathfrak{g}$ is a Lie algbera and Q is a CE differential seen as a vector field. If C^α denote coordinates on $\mathfrak{g}[1]$ then $QC^\alpha = -\frac{1}{2}U^\alpha_{\beta\gamma}C^\beta C^\gamma$. Denoting $\sigma^*(C^\alpha) = A^\alpha_a(x)\theta^a$ we get

$$d_X \circ \sigma^* = \sigma^* \circ Q \quad \Longrightarrow \quad dA + \frac{1}{2}[A, A] = 0$$

Gauge transformations:

$$\delta A = d\epsilon + [A, \epsilon]$$

Topological PDE. There exists a finite dimensional BV analog of diffiety. Example known from *AKSZ*

Presymplectic structures and BV quantization

Lagrangian induces presymplectic structure $\sigma \in \Omega^{(n-1,2)}(\mathcal{E})$ on the equation manifold.

Crnkovic, Witten, 1987, Hydon 2005, Khavkine 2012, Alkalaev M.G. 2013, Sharapov 2016

Def. Compatible presymplectic structure on gauge PDE $(E_{T[1]X}, Q)$ is a vertical 2-form ω on $E_{T[1]X}$ satisfying:

$$d\omega = 0\,, \qquad L_Q\omega = 0$$

Vertical forms are understood as equivalence classes

Defines "Hamiltonian" (or, better, covariant BRST charge) via

$$i_Q\omega = d\mathcal{H}\,, \qquad \mathrm{gh}(\mathcal{H}) = n$$

Intrinsic action

Action functional on the space of section of $(E_{T[1]X}, Q, \omega)$

$$S[\sigma] = \int_{T[1]X} (\sigma^*(\chi)(d_X) - \sigma^*(\mathcal{H}))$$

where χ is a presymplectic potential, i.e. $\omega = d\chi$. $\chi \to \chi + d\rho$ adds boundray term.

BV extension (AKSZ-type). Supersection $\widehat{\sigma}$:

$$S^{BV}[\widehat{\sigma}] = \int_{T[1]X} (\widehat{\sigma}^*(\chi)(d_X) - \widehat{\sigma}^*(\mathcal{H}))$$

If e.g. $\mathrm{gh}(C) = 1$ then $\sigma^*(C) = A_a(x)\theta^a$ while $\widehat{\sigma}^*(C) = \overset{0}{C}{}^a + A_a\theta^a + \overset{2}{\xi}_{ab}\theta^a\theta^b + \dots,$

Interpretation? What this has to do with the gauge PDE in question? *Alkalaev, MG 2013, MG 2016, MG, Kotov, ...*

Idea: assume ω regular and take a symplectic quotient. Does not always work in a naive way in interesting cases.

Refined idea: locally, sections are fiber-valued functions, take:

$$Smaps(T[1]X, F) = Smaps(X, Smaps(T_x[1]X, F))$$

$M = Smaps(T_x[1]X, F))$ is finite-dimensional provided F is. Natural lift of ω to M

$$\omega^M = \int d^n\theta \; \omega_{AB}(\psi(\theta))d\psi^A(\theta) \wedge d\psi^B(\theta)\,, \qquad \text{gh}(\omega^M) = -1$$

Now assume that ω^M is regular and take a symplectic quotient. We have arrived at BV formulation! With BV symplectic structure $\omega^M(dx)^n$ and BV master action S^{BV}!

State of the art: in "good" situations this BV is equivalent to the initial gauge theory provided ω arises from Lagrangian. For the moment only examples....

Scalar field

Usual PDE setting. $(\mathcal{E}, C)$ equipped with $\omega \in \bigwedge^{n-1,2}(\mathcal{E})$ such that $d_{\mathsf{h}}\omega = d_{\mathsf{v}}\omega = 0$. It follows that locally $d = \chi + l$ with $\chi \in \bigwedge^{n-1,1}(\mathcal{E})$ and $l \in \bigwedge^{n,0}(\mathcal{E})$. The intrincis action: *MG, 2016*

$$S[\sigma] = \int_X \sigma^*(\chi + l)$$

Take for instance scalar field:

$$L = \frac{1}{2}\eta^{ab}\phi_a\phi_b - V(\phi)$$

$\mathcal{E}$ is coordinatized by $x^a, \phi, \phi_a, \phi_{ab}, \ldots$ with $\phi_{abc\ldots}$ traceless.

$$d_{\mathsf{h}}x^a = dx^a\,, \quad d_{\mathsf{h}}\phi = dx^a\phi_a,, \quad d_{\mathsf{h}}\phi_a = dx^b(\phi_{ab} - \frac{1}{n}\eta_{ab}\frac{\partial V}{\partial\phi})\,, \quad \ldots$$

The presymplectic potential and 2-form:

$$\chi = (dx)^{n-1}_a\phi^a d_{\mathsf{v}}\phi\,, \quad \omega = (dx)^{n-1}_a d_{\mathsf{v}}\phi^a d_{\mathsf{v}}\phi$$

The Hamiltonian:

$$\mathcal{H} = (dx)^n(\phi_a\phi^a - L|_{\mathcal{E}}) = \frac{1}{2}\phi^a\phi_a + V(\phi)$$

The intrinsic Larangian: *Schwinger*

$$\mathcal{L}^c = (dx)^n\left(\phi^a(\partial_a\phi - \frac{1}{2}\phi_a) - V(\phi)\right)$$

What about symplectic structure?

$$\widehat{\sigma}^*(\phi) = \phi(x) + \theta^a\phi^1_a(x) + \ldots \qquad \widehat{\sigma}^*(\phi_a) = \phi_a(x) + \theta^b\phi^1_{a|b}(x) + \ldots$$

where $\mathrm{gh}(\phi^1_a) = \mathrm{gh}(\phi^1_{a|b}) = -1\,, \quad \mathrm{gh}(\phi) = \mathrm{gh}(\phi_a) = 0$. Only for these coordinates ω^M is nondenerate resulting in:

$$\omega^M = d\phi^0 \wedge d\phi^1_{a|a} + d\phi^0_a \wedge d\phi^1_a\,, \qquad \phi^1_{a|a} = \eta^{ab}\phi^1_{a|b}$$

Correct symplectic structure and set of variables for BV for this system!

Maxwell

Recall: $E_{T[1]X} = T[1]X \times M$, Fiber coordinates:

$$C, \quad \mathrm{gh}(C) = 1\,, \qquad F_{a|b}, \quad F_{a|b_1 b_2}, \quad \ldots \quad F_{a|b_1 \ldots b_l} \quad \ldots \qquad \mathrm{gh}(F_{\ldots}) = 0$$

$$Qx^a = \theta^a, \qquad Q\theta^a = 0, \qquad QC = \frac{1}{2} F_{ab}\theta^a\theta^b, \qquad QF_{a|b} = \theta^c F_{a|bc}, \qquad \ldots$$

Presymplectic structure *Alkalaev, M.G. 2013*

$\omega = (\theta)^{(n-2)}_{ab} F^{a|b} dC\,,$ indexes rised/lowered with Minkowski metric

Intrinsic action ($\sigma^*(C) = A_a(x)\theta^a, \sigma^*(F_{a|b}) = F_{a|b}(x)$):

$$S[\sigma] = \int d^n x (\partial_a A_b) F^{a|b} - \frac{1}{4} F_{a|b} F^{a|b}$$

Presymplectic structure on supermaps gives correct BV form!

$$\omega^M = dC \wedge \overset{2}{F}{}^{a|b}_{ab} + dA_a \wedge \overset{2}{F}{}^{a|b}_{b} + dF_{a|b} \wedge \overset{2}{C}_{ab}$$

Einstein gravity

In the example of gravity: *Alkalaev, M.G. 2013*

$$\chi = \frac{1}{2}\epsilon_{abcd} d\omega^{ab} e^c e^d\,, \qquad \sigma = d\omega^{ab} de^c \epsilon_{abcd} e^d$$

"Hamiltonian" (terms involving $W^{cd}_{ab|e}(x)$ vanish)

$$\mathcal{H} = Q^A \chi_A = -\frac{1}{2}\omega^a_c \omega^{cb}\epsilon_{abcd} e^c e^d$$

Intrinsic action (frame-like GR action):

$$S^C = \int \chi_A \boldsymbol{d}\psi^A - \mathcal{H} = \int (\boldsymbol{d}\omega^{ab} + \omega^a{}_c \omega^{cb})\epsilon_{abcd} e^c e^d$$

Familiar Cartan-Weyl action for GR. Generalization for general $n > 2$ and $\Lambda \neq 0$ is straightforward.

Just like in the case of scalar defines full BV on superjets *MG, Kotov, 2020*

Conclusions

- Gauge PDEs as geometric objects. Well suited to work with diffeomorphims-invariant and topological models. Notion of equivalence.

- Parent formulation – AKSZ formulation of generic gauge PDEs. Determines a "canonical" first-order realization in terms of a jet-bundle associated to the equation manifold

- Comprise "frame-like" formulation of the system. The respective free differential algebra arises from BRST differential. E.g. the Cartan-Weyl form of gravity arises from a minimal model of the BRST complex.

- The (BV) and the BV symplectic structure are encoded in the graded presympletic structure on the gaueg PDE.

- In the case of variational systems unifies Lagrangian and Hamiltonian BRST formalism, cf. BV/BFV approach of *Cattaneo et all.*

- Gives an invariant approach to study boundary values of gauge fields. In particular in the AdS/CFT correspondence context. *Bekaert, M.G. 2012*. In particular, Fefferman-Graham construction (and tractor calculus) can be seen as a certain gauge PDE. *M.G. 2012, M.G. Waldron 2011, Bekaert, M.G. Skvortsov 2017*

- Sucessful applications in constructing new models of HS theory, e.g. Type-B theory (AdS holographic dual to conformal spinor on the boundary) *M.G. Skvortsov 2018*

- Recent construction of Lagrangians for AdS_4 higher spin gravity in terms of presymplectic AKSZ. *Sharapov, Skvortsov 2020*