Hexagonal geodesic 3-webs

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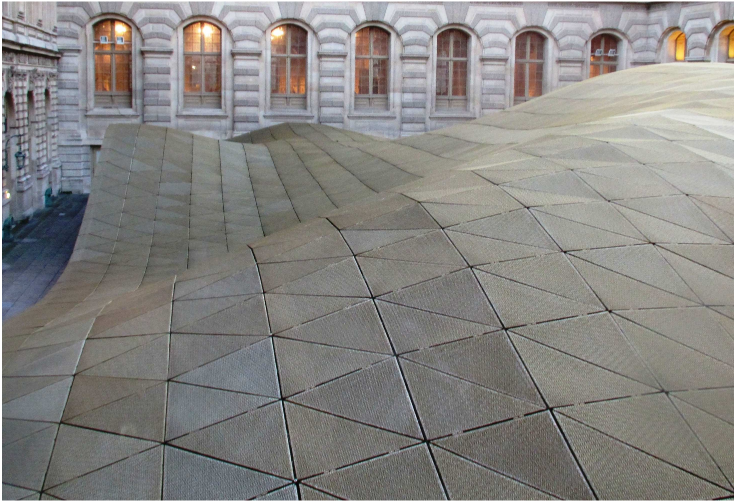
Freeform Architecture



Museum of Islamic Arts at Louvre

Equations for Metric

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$$ds^2 = E(u,v)du^2 + 2F(u,v)dudv + G(u,v)dv^2$$

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Lie Symmetry Algebra

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If $exp(tX)$ is not isometry then it is homothety.

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