Workshop on

Geometry of Differential Equations and Integrability

Hradec nad Moravicí, Czech Republic

October 11–15, 2010

Abstracts

Hynek Baran, *JETS, a way to empower calculations on differential equations in total derivatives on diffieties*

JETS is a tool to compute symmetries, conservation laws, zero-curvature representations, recursion operators, and many other invariants of systems of partial differential equations. A short review of JETS will be given along with a demonstration of some of its advanced capabilities.

Maciej Błaszak, *Bi-presymplectic theory of Stäckel systems*

Bi-presymplectic chains of one-forms of arbitrary co-rank are considered. The conditions in which such chains represent some Liouville integrable systems and the conditions in which there exist related bi-Hamiltonian chains of vector fields are presented. In order to derive the construction of bi-presymplectic chains, the notions of dual Poisson-presymplectic pair, d-compatibility of presymplectic forms and d-compatibility of Poisson bivectors is used. The completely algorithmic construction of separation coordinates is demonstrated. It is also proved that Stäckel separable systems have bi-inverse-Hamiltonian representation, i.e., are represented by bi-presymplectic chains of closed one-forms. The co-rank of related structures depends on the explicit form of separation relations.

Joseph Krasil'shchik, *On the tangent and cotangent coverings over differential equations. Part I: Computations*

I shall outline a simple scheme to compute recursion, Hamiltonian and symplectic operators for a wide class of PDEs. A number of examples will be discussed. Theoretical foundations of the scheme will be exposed in the talk by A. Verbovetsky.

Andrzej Maciejewski, *Necessary conditions for classical super-integrability of a certain family of potentials in constant curvature spaces*

We formulate the necessary conditions for the maximal super-integrability of certain family of classical potentials defined in the constant curvature two-dimensional spaces. We give examples of homogeneous potentials of degree -2 on Euclidean plane as well as their equivalents on sphere and the hyperbolic plane for which these necessary conditions are also sufficient. We show explicit forms of the additional first integrals which always can be chosen polynomial with respect to the momenta and which can be of an arbitrary high degree with respect to the momenta.

Luigi Martina, *Dynamics in the non-commutative plane and generalizations*

Some aspects of the "exotic" particle, associated with the two-parameter central extension of the planar Galilei group are reviewed. A fundamental property is that it has non-commuting position coordinates.

Other and generalized non-commutative models are also discussed. Minimal as well as anomalous coupling to an external electromagnetic field is presented. Supersymmetric extension is also considered. Exotic Galilean symmetry is also found in Moyal field theory. Similar equations arise for a semiclassical Bloch electron, used to explain the anomalous/spin/optical Hall effects.

Michal Marvan, *On new integrable classes of surfaces*

Recent classification of integrable classes of Weingarten surfaces along with new developments will be reviewed.

Giovanni Moreno, *A cohomological formalism for natural boundary conditions*

A free boundary variational problem admits a rich geometrical interpretatation in the framework of infinite jet spaces and related cohomological structures. In this picture, the so-called natural boundary conditions (or transversality conditions, depending on the context) appear spontaneously in a graded version of the Euler–Lagrange equations.

Oleg Morozov, *Lie pseudo-groups and zero-curvature representations of differential equations*

In this talk I will discuss the interplay between Cartan's structure theory of Lie pseudo-groups and zero-curvature representations of integrable (2+1) dispersionless equations.

Ziemowit Popowicz, *The conserved quantities for the generalized Riemann equation*

We analyze the integrability property of the generalized hydrodynamical Riemann type equation $D_t^N u = 0$ for arbitrary N. The infinite hierarchies of polynomial and non-polynomial conservation laws, both dispersive and dispersionless, are presented. Special attention is paid to the cases $N = 2, 3$ and $N = 4$, for which the conservation laws, Lax type representations and Hamiltonian structures are analyzed in detail. We also show that the case $N = 2$ is equivalent to a generalized Hunter–Saxton dynamical system, whose integrability follows from the results obtained. As a byproduct of our analysis we demonstrate a new set of non-polynomial conservation laws for the related Hunter–Saxton equation.

Maria Przybylska, *On certain classes of integrable and super-integrable homogeneous potentials*

We study the integrability of two-dimensional natural Hamiltonian systems with indefinite form of kinetic energy and homogeneous potentials of degree k of the form $H = 2p_1p_2 + q_1^l q_2^{k-l}$, where k, l are rational numbers. We show results of application of the direct method for search of first integrals. Three classes of integrable potentials are presented for that separations of variables are made and superintegrable cases are specified. In this aim some solvability results for hypergeometric equations are applied.

Vladimir Rubtsov, *Non-abelian Poisson structures*

We study some general non-abelian quadratic Poisson brackets and gave some classification results in the case of the free associative algebra with 2 generators. Relations with double Poisson structure of M. Van den Bergh and structures of W. Crowly–Boevy are discussed.

Péter Sebestyén, *On normal forms of irreducible $sl(n)$-valued zero curvature representations*

In this talk we recall the short history of development of normal forms of irreducible $sl(n)$-valued zero curvature representations. Some unpublished recent results will be presented and future plans discussed.

Artur Sergyeyev, *Reciprocal transformations and deformations of integrable hierarchies*

We present changes of variables that transform new integrable hierarchies found by Szablikowski and Błaszak using the R-matrix deformation technique [J. Math. Phys. 46 (2005), paper 042702, nlin.SI/0501044] into known Harry–Dym-type and mKdV-type hierarchies.

Błażej Szablikowski, *Classical r-matrix like approach to Frobenius manifolds and WDVV equations*

The theory of Frobenius manifolds was formulated by B. Dubrovin as a coordinate-free formulation of the WDVV associativity equations on a so-called prepotential function, appearing in the context of 2-dimensional topological field theories. A standard approach in the construction of Frobenius manifolds relies on the so-called Landau–Ginzburg model.

It is well known that the theory of Frobenius manifolds is closely related to the theory of bi-Hamiltonian hierarchies of hydrodynamic type. These systems together with their Hamiltonian structures are efficiently constructed by means of the so-called classical r-matrix formalism.

Our goal is to exploit the classical r-matrix formalism in order to build principal hierarchies of Frobenius manifolds in such a way that their Hamiltonians could be used to derive prepotential functions giving rise to associativity equations. We will present a method to construct Frobenius algebras, using the concept of double algebras, and show its relation to the linear Lie–Poisson brackets of the classical r-matrix formalism applied to Poisson algebras. Besides, we derive a simple recursion relation for the generating Hamiltonians of the corresponding principal hierarchies. We illustrate this in the context of the integrable hierarchies associated with dispersionless KdV and dispersionless Toda as well as show the connectionwith with the τ-function in the Whitham theory proposed by Krichever and Landau–Ginzburg models.

Alexander Verbovetsky, *On the tangent and cotangent coverings over differential equations. Part II: Invariance*

Tangent and cotangent coverings are analogs of the tangent and cotangent bundles in the category of differential equations. We shall discuss their invariant geometric definitions for the case of two-line equations.

Luca Vitagliano, *Hamilton–Jacobi field theory*

The Hamilton–Jacobi formalism for Lagrangian mechanical systems (in the Hamiltonian picture) can be extended to higher derivative Lagrangian field theory. I will show that Hamilton–Jacobi field theory gives rise to a secondary (i.e. functional) Hamilton-Jacobi theory on the covariant phase space, i.e., the space of solutions of Euler–Lagrange field equations.

Raffaele Vitolo, *CDIFF: a suite of REDUCE packages for computations in geometry of differential equations*

I will review the basic features of CDIFF, a set of REDUCE packages developed at the university of Twente. CDIFF has been succesfully used for computations of Hamiltonian operators. Recently, REDUCE became free software; this motivated me to write a user guide to CDIFF packages, complemented by several example programs. All this can be found at http://gdeq.org. I'm currently writing an interface to CDIFF which will also be discussed in my talk.