Formulae

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1 Still medium

$\Phi = \mathbb{R}\left(m_1, ..., m_k, V, E, S, \mu_1, .., \mu_k, p, T\right)$

$$\begin{aligned}
\omega &= dE - TdS + pdV - \sum_{i=1}^{k} \mu_i dm_i, \\
\Psi &= S - T^{-1}E, \\
L &: = \left\{E = T^2 \frac{\partial \psi}{\partial T}, p = T\frac{\partial \psi}{\partial V}, \mu_i = -T\frac{\partial \psi}{\partial m_i}, S = \psi + T\frac{\partial \psi}{\partial T}\right\}, \\
X_f &= -f_T \partial_S - \left(f + pf_p + Tf_T + \sum_{i=1}^{k} \mu_i f_{\mu_i}\right) \partial_E + f_p \partial_V - \\
& \sum_{i=1}^{k} f_{\mu_i} \partial_{m_i} + \left(f_E T + f_S\right) \partial_T + \left(f_E p - f_V\right) \partial_p + \sum_{i=1}^{k} (f_E \mu_i + f_{m_i}) \partial_{\mu_i}.
\end{aligned}$$

1.1 Densities

$\widetilde{\Phi}=\mathbb{R}\left(\rho_1,...,\rho_k,e,\mu_1,..,\mu_k,p,T\right)$

$$
\begin{aligned}
s &= \frac{S}{V}, e=\frac{E}{V}, \rho_1=\frac{m_1}{V}, ..., \rho_k=\frac{m_k}{V}, T, p, \mu_1, ..., \mu_k, \\
\widetilde{\omega} &= sdT-\ de-\sum_{i=1}^{k}\rho_i d\mu_i,\ e=Ts-p+\sum_{i=1}^{k}\rho_i\mu_i. \\
\widetilde{L} &= \left\{\varepsilon=T^2\psi_T, \mu_i=-T\psi_{\rho_i}, p=T\psi-\sum_{i=1}^{k}\rho_i\psi_{\rho_i}\right\} \\
\widetilde{X}_f &= -T\left(Tf_T+(\varepsilon+p)f_p+\sum_{i=1}^{k}\mu_i f_{\mu_i}\right)\partial_\varepsilon-\sum_{i=1}^{k}\left(f_{\mu_i}+f_p\rho_i\right)\partial_{\rho_i}+f_\varepsilon T^2\partial_T \\
&\quad +\left((p+\varepsilon)f_p+\sum_{i=1}^{k}\rho_i f_{\rho_i}+f\right)\partial_p+\sum_{i=1}^{k}\left(f_{\rho_i}+f_\varepsilon\mu_i\right)\partial_{\mu_i}. \\
\varkappa &= \sum_{i,j=1}^{k}\psi_{\rho_i\rho_j}d\rho_i\cdot d\rho_j-\left(\psi_{TT}+\frac{2\psi_T}{T}\right)dT\cdot dT.
\end{aligned}
$$

2 Moving medium

$\Gamma=\Phi\times\mathrm{End}\left(T_aM\right)\times\mathrm{End}\left(T_a^*M\right)$

Coordinates $\widetilde{\Gamma}\ \left(\rho_1,...,\rho_k,\varepsilon,\Delta,\mu_1,..,\mu_k,p,T,\Sigma\right)$

$\mathbb{H}\subset\mathrm{SO}\left(\dim M\right)-$ the holonomy group.

$$
\begin{aligned}
\omega &= dS-T^{-1}dE-T^{-1}\,\mathrm{Tr}\left(\Sigma^* d\Delta\right)-T^{-1}pdV+\sum_{i=1}^{k}T^{-1}\mu_i dm_i, \\
\widetilde{L} &: =\{\varepsilon=T^2\frac{\partial\psi}{\partial T},\ \mu_i=-T\frac{\partial\psi}{\partial\rho_i}, \Sigma=T\frac{\partial\psi}{\partial\Delta}, \\
p &= T\psi^c-T\sum_{i=1}^{k}\rho_i^c\frac{\partial\psi^c}{\partial\rho_i^c}\}.
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{X}_f &= -T\left(Tf_T + (\varepsilon + p) f_p + \sum_{i=1}^{k} \mu_i f_{\mu_i} + \Sigma f_\Sigma\right)\partial_\varepsilon \\
&\quad - \sum_{i=1}^{k}\left(f_{\mu_i} + f_p \rho_i\right)\partial_{\rho_i} - \left(f_\Sigma + f_p \Delta\right)\partial_\Delta + f_\varepsilon T^2 \partial_T \\
&\quad + \left((p+\varepsilon) f_p + \sum_{i=1}^{k} \rho_i f_{\rho_i} + \Delta f_\Delta + f\right)\partial_p + \sum_{i=1}^{k}\left(f_{\rho_i} + f_\varepsilon \mu_i\right)\partial_{\mu_i} + \left(f_\Delta + f_\varepsilon \Sigma\right)\partial_\Sigma . \\
\varkappa &= \sum_{i,j=1}^{k} \psi_{\rho_i \rho_j} d\rho_i \cdot d\rho_j - \left(\psi_{TT} + \frac{2\psi_T}{T}\right) dT \cdot dT + \frac{\partial^2 \psi}{\partial \Delta^2} d\Sigma \cdot d\Sigma .
\end{aligned}
$$

$\psi\left(\rho_1, ..., \rho_k, \varepsilon, \Delta\right)$ and $f\ \left(\rho_1, ..., \rho_k, \varepsilon, \Delta, \Sigma, T, p, \mu_1, ..., \mu_k\right)$ are$-\mathbb{H}$ invariants=depend on $\mathbb{H}$ -invariants in $\operatorname{End}\left(T_a\right)$.

3 N-S equations

1. Conservation of momentum:

$$
\left(\frac{\partial X}{\partial t} + \nabla_X (X)\right) = \operatorname{div}^\flat \left(T \frac{\partial \psi}{\partial \Delta}\right).
$$

2. Conservation of mass:

$$
\begin{aligned}
\frac{\partial \rho_i}{\partial t} + \rho_i \operatorname{div}\left(\rho_i X\right) + f_{\mu_i} + f_p \rho_i &= 0, \\
i &= 1, .., k.
\end{aligned}
$$

3. Heat conduction:

$$
\frac{\partial T}{\partial t} + X(T) - a\Delta_g (T) - f_\varepsilon T^2 = 0.
$$